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A Generalized Scalarizing Problem of Multicriteria Optimization

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Abstract: *The paper describes a generalized classification - based scalarizing problem of multicriteria optimization. The basic properties of the generalized scalarizing problem are discussed and it is shown that some of the known scalarizing problems of the reference point and the classification based scalarizing problems as well can be obtained from it with the help of different transformations.*

Keywords: *multicriteria optimization, scalarizing problem.*

1. Introduction

The problems of multicriteria decision making can be divided in two separate classes depending on their formal statement (S t e u e r [11], V i n c k e [15]). A finite number of explicitly set constraints in the form of functions defines an infinite number of feasible alternatives in the first class. These problems are called problems of multicriteria optimization (MO). A finite number of alternatives is explicitly set in the second class of problems. These problems are called problems of multicriteria analysis (MA).

Several criteria (objective functions) are simultaneously optimized in the feasible set of solutions (alternatives) in problems of MO. In the general case there does not exist one solution only, which optimizes the criteria. There exists, however, a set of solutions in the variables space and a respective set in the criteria space, which is characterized by the following: each improvement in the value of one criterion leads to deterioration in the value of one other criterion at least. These sets are called Pareto optimal sets. The variables set is called also an efficient set, and the criteria set – a non-dominated set. Every element of these sets could be a solution of the multicriteria optimization problem. In order to select a given element, additional information is necessary that is supplied by the so-called decision maker (DM). The information, which the DM sets, reflects his/her global preferences with respect to the quality of the solution obtained.

The problems of MO are usually solved by scalarization (H w a n g and M a - s u d [3], S a w a r a g i, N a k a y a m a and T a n i n o [9], S t e u e r [11], M i e t t i- n e n [4]). Scalarization means transformation of the multicriteria optimization problem into one or several single-criterion problems. This transformation enables the use of the theory and methods of single-criterion optimization. The basis of such transformation lies in the fact that each Pareto optimal solution of the multicriteria optimization problem can be obtained as an optimal solution of the scalarizing problem. One single-criterion scalarizing problem must satisfy the following two conditions, in order to be a scalarizing problem of the multicriteria optimization problem:

– each Pareto optimal solution of the multicriteria optimization problem has to be found altering the values of the scalarizing problem parameters;

– appropriate values of the parameters of the scalarizing problem should be found for each Pareto optimal solution of the multicriteria optimization problem in such a way, that its optimal solution be exactly this Pareto optimal solution of the multicriteria problem.

The DM (DM is either one person or a group of persons with similar concepts) can express his/her preferences among the separate Pareto optimal solutions with the help of the parameters values. The DM has to select the final (most preferred) solution and is responsible for this choice.

MO methods apply in a different way (M i e t t i n e n [4]) different scalarizing problems such as: the problem of the weighed sum; the problem of e-constraints; the problem of the reference point; the classification based problems, etc. Though the scalarizing problems used determine to a large extent the type of these methods, the references suggest comparative analysis of the methods rather than comparison of the different scalarizing problems (M i e t t i n e n and M a k e l a [6]). In an analogous way some attempts to unite different methods (Gardiner and Steuer, 1994), are presented more often than attempts to generalize different scalarizing problems.

The present paper describes a generalized scalarizing problem, from which some of the more famous scalarizing problems of the reference point and of the classification based scalarizing problems can be obtained, altering given parameters. The generalized scalarizing problem connects the separate scalarizing problems to a high degree and thus it can be said that this problem combines their features as well. A generalized MO method can be developed on its basis, which joins extensively the features of the separate methods that apply the respective scalarizing problems.

The paper is organized as follows. The second chapter presents a general statement of the multictriteria optimization problem and some definitions are given. The third chapter describes the generalized scalarizing problem and some of its properties are proved. The transformation of this problem into well-known scalarizing problems is shown in the fourth part. The conclusion is given in the fifth chapter.

2. Formal statement of the problem of multicriteria optimization

The general problem of multicriteria optimization can be stated as follows:

(1) $$\text{“max”}\ \{f_k(x)\},\ k \in K,$$

under the condition

(2) $$x \in X,$$

where $f_k(x)$, $k\in K=\{1, 2,\ldots, p\}$ are different criteria (objective functions) $f_k:\mathbf{R}^n\to\mathbf{R}$, which have to be simultaneously maximized; $x = \{x_1, ..., x_j, ..., x_n)^{\mathrm{T}}$ is the vector of the variables, belonging to the non-empty feasible set $X\in\mathbf{R}^n$; $Z = f(X)\subset \mathbf{R}^p$ is the feasible set of the criteria values.

The following definitions will be introduced for greater clarity:

Definition 1. The solution $x\in X$ is called a Pareto optimal solution of the multicriteria optimization problem if there does not exist another $\bar{x}\in X$, for which the condition below given will be satisfied:

$$f_k(\bar{x}) \geq f_k(x),\ k\in K,$$

and $f_k(\bar{x}) > f_k(x)$ for at least one index $k\in K$.

Definition 2. The solution $x\in X$ is called a weak Pareto optimal solution of the multicriteria optimization problem if there does not exist another $\bar{x}\in X$, for which the following condition is satisfied:

$$f_k(\bar{x}) > f_k(x),\ \ k\in K.$$

Definition 3. The vector $z = f(x)=(f_1(x),..., f_p(x))^{\mathrm{T}}$; $z = f(x)\in Z$ is called a (weak) Pareto optimal solution in the criteria space, if $x\in X$ is a (weak) Pareto optimal solution in the variables space.

Definition 4. The vector $z^*\in Z$ is called an ideal vector, if its every component z_k, $k = \overline{1,p}$ is obtained by individual optimization of the criterion f_k on its feasible set X.

Definition 5. The vector $z^{**}\in Z$ is called a utopia vector, if its every component z_k^{**}, $k = \overline{1,p}$ satisfies the following condition:

$$z_k^{**} = z_k^{*} + \varepsilon_k\ ,$$

where ε_k is a small positive number.

Definition 6. The components of nadir vector z^{nad} are the low limits of Pareto optimal set. It is usually difficult to obtain the exact values of these components, but they can be evaluated approximately by the pay-off table (M i e t t i n e n [4]).

Definition 7. The current preferred solution $z = (f_1,..., f_k, ..., f_p)$ is a weak Pareto optimal or Pareto optimal solution in the ctiteria space, selected by the DM at the current iteration.

Definition 8. A reference point (Wierzbicki (1980)) or a reference vector $\bar{z} = (\bar{z}_1,..., \bar{z}_p)^{\mathrm{T}}$ is called that vector, the criteria values of which are equal to the desired or aspiration values set by the DM. These aspiration values may be achieved or not.

Definition 9. Classification or partition of the criteria (B e n a y o u n et a l. [2], N a k a y a m a [7], N a r u l a and V a s s i l e v [8], M i e t t i n e n [4]) is called the implicit partition of the criteria into classes accomplished by the DM with relation to what changes of the criteria values the DM wishes to obtain, compared to the corresponding criteria values at the current solution.

3. Generalized scalarizing problem

In order to obtain a (weak) Pareto optimal solution of the multicriteria optimization problem, starting directly or indirectly from the current (weak) Pareto optimal solution found, the following scalarizing problem **A** could be applied:

Minimize

(3) $S(x)=\max(\max_{k\in K^{\geq}}(F_k^1 - f_k(x))/G_k^1, \max_{k\in K^{<}\cup K^{\leq}}(F_k^2 - f_k(x))/G_k^2) R \max_{k\in K^{>}}(F_k^3 - f_k(x))/G_k^3$

under the constraints

(4) ,

(5) $f_k(x) \geq f_k - D_k, k \in K^{\leq}$,

(6) ,

(7) ,

(8) $x\in X$,

where are constants;

– t_k^- and t_k^+ are the upper and lower limit of the desired interval of change in the values of the criterion with an index k;

– D_k is the maximal value of deterioration of the criterion with an index k;

– R is equal to the arithmetic operation + or a separating symbol , ;

– $K^{\geq}$ is the set of the criteria, the values of which the DM wants to improve to the aspiration values;

– $K^{>}$ is the set of the criteria, the values of which the DM wants to improve;

– $K^{\leq}$ is the set of the criteria that the DM agrees to be deteriorated to be deteriorated by a maximal value D_k;

– $K^{=}$ is the set of the criteria the values of which the DM wants to preserve;

– $K^{<>}$ is the set of the criteria the values of which the DM wishes to be within defined intervals;

– K^0 is a set of criteria the DM is not interested in.

Theorem 1. The optimal solution of the scalarizing problem **A** is a weak Pareto optimal solution of the multicriteria optimization problem.

P r o o f. Let $K^{\geq}$ and/or $K^{>} \neq \varnothing$ and x^* be an optimal solution of the scalarizing problem **A**. Then the following condition is satisfied:

(9) $S(x^*) \leq S(x), x\in X$,

and constraints (4)–(7) are satisfied for x^*.

Let us assume that x^* is not a weak Pareto optimal solution of the multicriteria optimization problem. Then there must exist $x'\in X$, for which the following conditions are satisfied:

(10) $f_k(x') > f_k(x^*), \quad k\in K$,

and constraints (4)-(8) are fulfilled for x'.

After a transformation of the objective function (3), using inequalities (10), the following inequality is obtained:

(11) $S(x')=\max(\max_{k\in K^{\geq}}(F_k^1 - f_k(x'))/G_k^1, \max_{k\in K^{<}\cup K^{\leq}}(F_k^2 - f_k(x'))/G_k^2) R \max_{k\in K^{>}}(F_k^3 - f_k(x')/G_k^3 =$

$$= \max(\max_{k\in K^{\geq}}((F_k^1 - f_k(x^*)) + (f_k(x^*) - f_k(x')))/G_k^1,$$

$$\max_{k\in K^{<}\cup K^{\leq}}((F_k^2 - f_k(x^*)) + (f_k(x^*) - f_k(x')))/G_k^2) \times$$

$$\times R\max_{k\in K^{>}}((F_k^3 - f_k(x^*)) + (f_k(x^*) - f_k(x'))) / G_k^3) <$$

$$< \max(\max_{k\in K^{\geq}}(F_k^1 - f_k(x^*)) / G_k^1, \max_{k\in K^{<}\cup K^{\leq}}(F_k^2 - f_k(x^*)) / G_k^3) R \max_{k\in K^{>}}(F_k^3 - f_k(x^*)) / G_k^3 = S(x^*).$$

From (11) and the definition of R, it follows that $S(x')< S(x^*)$, which is in contradiction with (9). Hence x^* is a weak Pareto optimal solution of the multicriteria optimization problem.

A (weak) Pareto optimal solution of the multicriteria optimization problem is found with the help of scalarizing problem **A**. If the wish is to obtain Pareto optimal solutions only, scalarizing problem **B** could be solved, that has the form:

Minimize

$$(12)\; T(x) = \max(\max_{k\in K^{\geq}}(F_k^1 - f_k(x)) / G_k^1, \max_{k\in K^{<}\cup K^{\leq}}(F_k^2 - f_k(x)) / G_k^2) R \max_{k\in K^{>}}(F_k^3 - f_k(x)) / G_k^3 +$$

$$+ \rho\left(\sum_{k\in K^{\geq}}(F_k^1 - f_k(x)) + \sum_{k\in K^{<}\cup K^{\leq}}(F_k^2 - f_k(x)) + \sum_{k\in K^{>}}(F_k^3 - f_k(x))\right)$$

under constraints (4)–(8) and ρ – a small positive number.

Theorem 2. The optimal solution of scalarizing problem **B** is a Pareto optimal solution of the multicriteria optimization problem.

P r o o f. Let $K^{\geq}$ and/or $K^{>} \neq \varnothing$, and x^* be an optimal solution of scalarizing problem **B**. Then the condition will be satisfied:

(13) $$T(x^*) < T(x),\; x\in X,$$

and constraints (4)–(7) are fulfilled for x^*.

Let us assume that x^* is not a Pareto optimal solution of the initial multicriteria optimization problem. Then there must exist $x'\in X$, for which the conditions below given are fulfilled:

(14) $$f_k(x') \geq f_k(x^*), \quad k\in K,$$

and $f_k(x') > f_k(x^*)$, for at least one index $k\in K$, and constraints (4)-(8) are satisfied for x'.

After a transformation of the objective function $T(x')$ of scalarizing problem **B**, using inequalities (4), the following inequality is obtained:

$$(15)\; T(x') = \max(\max_{k\in K^{\geq}}(F_k^1 - f_k(x')) / G_k^1, \max_{k\in K^{<}\cup K^{\leq}}(F_k^2 - f_k(x')) / G_k^2) R \max_{k\in K^{>}}(F_k^3 - f_k(x')) / G_k^3 +$$

$$+ \rho\left(\sum_{k\in K^{\geq}}(F_k^1 - f_k(x')) + \sum_{k\in K^{<}\cup K^{\leq}}(F_k^2 - f_k(x')) + \sum_{k\in K^{>}}(F_k^3 - f_k(x'))\right) =$$

$$= \max(\max_{k\in K^{\geq}}(F_k^1 - f_k(x^*)) + (f_k(x^*) - f_k(x')) / G_k^1,$$

$$\max_{k\in K^{<}\cup K^{\leq}}((F_k^2 - f_k(x^*) + (f_k(x^*) - f_k(x')) / G_k^2 \times$$

$$\times R\max_{k\in K^{>}}((((F_k^3 - f_k(x^*)) + (f_k(x^*) - f_k(x')) / G_k^3 +$$

$$+\rho\left(\sum_{k\in K^{\geq}}((F_k^1-f_k(x^*))+(f_k(x^*)-f_k(x')))\right)+\sum_{k\in K^{<}\cup K^{\leq}}((F_k^2-f_k(x^*))+(f_k(x^*)-f_k(x')))+$$

$$+\sum_{k\in K^{>}}((F_k^3-f_k(x^*))+(f_k(x^*)-f_k(x')))) < \max(\max(F_k^1-f_k(x^*))/G_k^1,$$

$$\max_{k\in K^{<}\cup K^{\leq}}(F_k^2-f_k(x^*))/G_k^2)R\max_{k\in K^{>}}(F_k^3-f_k(x^*))/G_k^3+$$

$$+\rho\left(\sum_{k\in K^{\geq}}(F_k^1-f_k(x^*))+\sum_{k\in K^{<}\cup K^{\leq}}(F_k^2-f_k(x^*))+\sum_{k\in K^{>}}(F_k^3-f_k(x^*))\right)=T(x^*).$$

From (15) and from the definition of R it follows that $T(x') < T(x^*)$, which contradicts to (13). Hence x^* is a Pareto optimal solution of the multicriteria optimization problem.

4. Transformation of the generalized scalarizing problem into known scalarizing problems

The scalarizing problems **A** and **B** are a generalization of the more or less wide- spread classification based scalarizing problems (B e n a y o u n et a l. [2], N a r u l a and V a s s i l e v [8], M i e t t i n e n and M a k e l a [5], V a s s i l e v et a l. [13], Vassileva et a l. [14]) and of the scalarizing problems of the reference point (W i e r z b i c k i [16], N a k a y a m a and S a w a r a g i [7], B u c h a n a n [1]). The reference point contains the aspiration levels of the criteria, which the DM wants or agrees to achieve: During the classification, the DM defines what changes of the criteria values with respect to their values at the current solution are desired or acceptable for him/her. These two ways of defining DM's preferences are relatively close and could be combined. Further on it will be demonstrated how some of the already familiar scalarizing problems can be obtained from scalarizing problem **A**.

4.1. STEM scalarizing problem

This scalarizing problem is used in STEM method (B e n a y o u n et a l. [2], V a n d e r p o o t e n and V i n c k e [12]). The DM is supposed to classify the criteria in two groups: $K^{\leq}$ and $K^{>}$. More precisely, he/she determines which criteria he agrees to be deteriorated and the maximal value of deterioration for each one of them. The remaining criteria have to be improved. STEM scalarizing problem has the following form:

Minimize

$$\max_{k\in K^{>}}\left(\frac{e_k}{\sum_{j\in K^{>}} e_j}(z_k^*-f_k(x))\right)$$

under constraints

$$f_k(x)\geq f_k-\Delta_k,\ k\in K^{\leq},$$

$$f_\kappa(x) \geq f_k,\ k \in K^{>},$$
$$x \in X,$$

where $e_k = \dfrac{z_k^* - z_k^{\mathrm{nad}}}{\max(|z_k^{\mathrm{nad}}|, |z_k^*|)}$.

Δ_k – maximal value of deterioration for the criterion with an index k.

The scalarizing problem STEM can be obtained from scalarizing problem **A** with the help of the following replacements:

1) $K^{<} = K^{\geq} = K^{=} = K^{><} = K^{0} = \varnothing$;
2) $D_k = \Delta_k$;
3) $F_k^3 = z_k^*$;
4) $G_k^3 = \dfrac{e_k}{\sum_{j \in K^{>}} e_j}$.

4.2. STOM scalararizing problem

STOM scalarizing problem is applied in the method of the satisfying compromise (N a k a y a m a and S a w a r a g i [7]). The DM must set the aspiration levels of the criteria in the reference point $\bar{z}$. After comparison of these aspiration levels of the criteria with their corresponding values at the current solution, the criteria can be divided into three groups: $K^{\geq}$, $K^{\leq}$ and $K^{=}$. Different statements of STOM scalarizing problem are used, one of them being:

Minimize

$$\max\left(\max_{k \in K^{\geq}} \frac{z_k^{**} - f_k(x)}{z_k^{**} - \bar{z}_k}, \max_{k \in K^{\leq}} \frac{z_k^{**} - f_k(x)}{z_k^{**} - \bar{z}_k} \right)$$

under the conditions

$$f_k(x) > f_k,\ k \in K,$$

$$x \in X,$$

where $\bar{z}_k$ is the k-th component of the reference point, z_k** – the k-th component of the utopia point.

STOM scalarizing problem can be obtained from scalarizing problem **A** by the replacements:

1) $K^{>} = K^{<} = K^{><} = K^{0} = \varnothing$;
2) $F_k^1 = F_k^2 = z_k^{**}$;
3) ;
4) $D_k = \propto$.

4.3. Scalarizing problem of the reference point

This scalarizing problem is used in the reference point method (W i e r z b i c k i [16]). The DM has to set the aspiration levels of the criteria at the reference point $\bar{z}$. Comparing these aspiration levels of the criteria with their respective values at the current solution, the criteria can be divided in two groups: $K^{\geq}$ and $K^{\leq}$. Then the statement of the scalarizing problem has the following form:

Minimize

$$\max\left(\max_{k\in K^{\geq}}\left(\frac{\bar{z}_k - f_k(x)}{|\bar{z}_k|}\right), \max_{k\in K^{\leq}}\left(\frac{\bar{z}_k - f_k(x)}{|\bar{z}_k|}\right)\right)$$

under the condition $x\in X$.

The scalarizing problem of the reference point can be obtained from scalarizing problem **A** after the following replacements:

1) $K^{>} = K^{<} = K^{=} = K^{><} = K^{0} = \varnothing$;

2) $F_k^1 = F_k^2 = \bar{z}_k$;

3) ;

4) $D_k = \infty$.

4.4. GUESS scalarizing problem

GUESS scalarizing problem is applied in GUESS method (B u c h a n a n [1]). The DM has to determine the components of nadir and of the reference point – z_k^{nad} and $\bar{z}$. After the comparison of the criteria values at the reference point and at the current solution, the criteria may be divided in two groups: $K^{\geq}$ and $K^{\leq}$. One of the statements of this scalarizing problem has the type:

Minimize:

$$\max\left(\max_{k\in K^{\geq}}\frac{(z_k^{\text{nad}} - f_k(x))}{\bar{z}_k - z_k^{\text{nad}}}, \max_{k\in K^{\leq}}\frac{(z_k^{\text{nad}} - f_k(x))}{\bar{z}_k - z_k^{\text{nad}}}\right)$$

under the condition $x\in X$ and the aspiration levels be not smaller than the corresponding values of the criteria at nadir point.

GUESS scalarizing problem can be obtained from scalarizing problem **A** after the following replacements:

1) $K^{>} = K^{<} = K^{=} = K^{><} = K^{0} = \varnothing$;

2) $F_k^1 = F_k^2 = z_k^{\text{nad}}$;

3) $G_k^1 = G_k^2 = \dfrac{1}{\bar{z}_k - z_k^{\text{nad}}}$;

4) $D_k = \infty$.

4.5. Scalarizing problem of the modified reference point

The scalarizing problem of the modified reference point is applied in the reference direction method (V a s s i l e v et a l. [13]). The DM has to define the aspiration levels of the criteria at the reference point $\bar{z}$. Comparing these aspiration levels of the criteria with their respective values at the current solution, the criteria can be divided in three groups: $K^{\geq}$, $K^{\leq}$ and $K^{=}$. The statement of this scalarizing function has the following form:

Minimize

$$\max\left(\max_{k\in K^{\geq}}\left(\frac{\bar{z}_k - f_k(x)}{\bar{z}_k - f_k}\right), \max_{k\in K^{\leq}}\left(\frac{f_k - f_k(x)}{f_k - \bar{z}_k}\right)\right)$$

under the conditions:

$$f_\kappa(x) \geq f_k,\ k\in K^{=},$$

$$x\in X,$$

where f_k is the value of the criterion with an index $k\in K$ in the current preferred solution.

The scalarizing problem of the modified reference point can be obtained from scalarizing problem **A** after the replacements:

1) $K^{>} = K^{<} = K^{><} = K^{0} = \varnothing$;

2) $F_k^1 = \bar{z}_k$;

3) $F_k^2 = f_k$;

4) ;

5) ;

6) $D_k = \propto$.

4.6. NIMBUS scalarizing problem

NIMBUS scalarizing problem is applied in the classification based method NIMBUS (M i e t t i n e n and M a k e l a [6]). The DM must divide the criteria in five groups: $K^{>}$, $K^{\geq}$, $K^{=}$, $K^{\geq}$, $K^{\leq}$ and K^{0} depending on what alterations of the criteria values he wishes in relation to their respective values at the current solution. Several versions of the scalarizing problem are applied in the method, one of them having the following form:

Minimize

$$\max\left(\max_{k\in K^{\geq}}\frac{(\bar{z}_\kappa - f_k(x))}{\left|z_k^*\right|}, \max_{k\in K^{>}}\frac{(z_k^* - f_k(x))}{\left|z_k^*\right|}\right)$$

under the conditions

, $k\in$,

$, k \in K^{\geq},$

$,$

$x \in X.$

NIMBUS scalarizing problem can be obtained from scalarizing problem **A**, replacing:

1) $K^{<} = K^{><} = \varnothing$;

2) $F_k^1 = z^*_k$;

3) $F_k^2 = \bar{z}_k$;

4)) $D_k = \Delta_k$;

5) ;

6) $R =$,.

4.7. Classification based scalarizing problem

This classification based scalarizing problem is used in the method described by V a s s i l e v a et a l. [14]. The DM possesses a wide range of setting his/her preferences for the desired changes of the criteria values in comparison with the criteria values at the current solution. On the basis of these preferences, the criteria may be divided into seven or less than seven classes. The scalarizing problem has the type:

Minimize

$$\max\left(\max_{k \in K^{\geq}} \frac{\left(\bar{z}_k - f_k(x)\right)}{\left|f_k^{'}\right|}, \max_{k \in K^{<} \cup K^{\leq}} \frac{(f_k - f_k(x))}{\left|f_k^{'}\right|} + \max_{k \in K^{>}} \frac{(f_k - f_k(x))}{\left|f_k^{'}\right|} \right)$$

under the conditions

$$f_k(x) \geq f_k,\ k \in$$

$$f_k(x) \geq f_k - \Delta_k, k \in K^{\leq},$$

$$f_k(x) \geq f_k - t_k^{-}, k \in K^{><},$$

$$f_k(x) \leq f_k + t_k^{+}, k \in K^{><},$$

$$x \in X,$$

where

$$f_k^{'} = \begin{cases} \varepsilon, \text{if } \left|f_k^{'}\right| \leq \varepsilon, \\ f_k, \text{otherwise} \end{cases}$$

and ε is a small positive number.

This scalarizing problem can be obtained from scalarizing problem **A** after the following replacements:

1) $F_k^1 = \bar{z}_k$;
2) $F_k^2 = f_k$;
3) $F_k^3 = f_k$;

4) ;

5) $D_k = \Delta_k$;
6) $R = +$.

The seven scalarizing problems above discussed, which can also be obtained from the generalized scalarizing problem with the help of different transformations, ensure (weak) Pareto optimal solutions only. Each one of these scalarizing problems has an modifications (M i e t t i n e n [4]), which facilitates the obtaining of Pareto optimal solutions. These scalarizing problems can be obtained from the generalized scalarizing problem **B** with the help of different transformations.

5. Conclusion

The scalarizing problems are an important part of the interactive methods solving multcriteria optimization problems. The scalarizing problems of the reference point and the classification based scalarizing problems have found wide application. The scalarizing problem described in the present paper is a generalization of the known scalarizing problems of the reference point and of the classification based sclalarizing problems. One of the advantages of the scalarizing problem discussed is that it may serve as a basis for the construction of a scalarizing problem with apriori set properties. Another advantage is that this problem only could be built in the design of software systems solving multicriteria optimization problems instead of the alternative of including many scalarizing problems. A number of other famous scalarizing problems can be dynamically obtained from it.

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Обобщена скаларизираща задача на многокритериалната оптимизация

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Статията описва обощена класификационно базирана скаларизираща задача на многокритериалната оптимизация. Разглеждат се основните свойства на обобщената скаларизираща задача и е показано, че някои от известните скаларизиращи задачи на отправната точка и класификационно базираните скаларизиращи задачи могат да се получат от нея с помощта на различни трансформации.