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A Learning-Oriented Method of Linear Mixed Integer Multicriteria Optimization*

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Abstract: *A learning-oriented interactive method for solving linear mixed integer problems of multicriteria optimization is described in the paper. The method offers the decision maker (DM) flexibility to express his/her preferences. The DM is encouraged, especially in the learning phase or when solving large problems, to solve continuous scalarizing problems or integer scalarizing problems approximately at many iterations. The method is realized in an experimental software system supporting the solution of linear mixed integer multicriteria optimization problems.*

Keywords: *linear mixed integer multicriteria optimization, interactive methods, optimization software systems.*

1. Introduction

The interactive methods are the most widely spread methods solving multicriteria optimization problems. Each iteration of such a method consists of two phases: a computational phase and a dialogue phase. In the computational phase one or more (weak) non-dominated or non-dominated solutions are generated with the help of a scalarizing problem. In the dialogue phase these (weak) non-dominated or non-dominated solutions are presented for evaluation to the decision maker (DM). In case the DM does not approve any of these solutions as a final solution (the most preferred solution) of the output multicriteria problem, he offers some information concerning his/her local preferences with the purpose to improve these solutions. This information is used to formulate a new scalarizing problem solved at the next iteration.

The interactive methods can be divided in two other main groups [2, 16]: search-oriented methods and learning-oriented methods. In the first group of methods a con-

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verging sequence of solutions is presented to the DM, supposing that he is undeviating in his preferences. These methods are convergent from a mathematical viewpoint. In the second group of methods the DM has the possibility to observe freely the elements of Pareto set. It is assumed in them, that the DM can choose by tests and errors the most preferred according to him solution. These methods are not convergent from a mathematical viewpoint, but they are convergent from the viewpoint of the DM – this is the so-called behavioural or intuitive convergence.

The quality of each interactive method is defined to a great extent by the quality of the dialogue with the DM. On its side, the quality of the dialogue with the DM depends on the type of information, which he sets in order to improve the local preferred (weak) non-dominated or non-dominated solution, on the time for scalarizing problem solution, on the possibilities for DM's learning with respect to the multicriteria problem being solved, on the type and number of the compared new (weak) non-dominated or non-dominated solutions with respect to the local preferred solution.

Linear programming problems are used as scalarizing problems in solving linear problems of multicriteria optimization. These problems are easy solved problems. Hence, in the interactive methods for solving multicriteria linear problems, the time for the scalarizing problems solution does not play a significant role. In the development of these methods main attention is paid to the type of information, which the DM may set in order to improve a local preferred (weak) non-dominated or non-dominated solution.

In a larger part of the interactive methods solving linear problems of multicriteria optimization [10, 22] that are well-known, the aspiration levels of the criteria, which the DM wishes to achieve, are used mainly as such information. The aspiration levels define in the criteria space the so-called local reference point, starting point, etc. These interactive methods utilize scalarizing problems, belonging to the group of the reference point scalarizing problems. The classification-oriented scalarizing problems have been included much more rarely in the interactive methods solving linear multicriteria problems up to now. The possibility to learn the DM while solving the linear multicriteria problem is another significant feature of the interactive methods. Besides DM's freedom to move in the non-dominated set, these possibilities are also expressed in defining more than one (weak) non-dominated or non-dominated solutions during the computational phase. These solutions are presented to the DM for evaluation [8].

When solving linear integer problems of multicriteria optimisation, integer programming problems are used as scalarizing problems, which are NP-hard problems [3]. The exact algorithms for their solution have exponential complexity and even finding a feasible integer solution is so difficult as finding an optimal solution. That is why in the development of interactive methods for solving multicriteria linear integer problems, it is obligatory to take into account the time for scalarizing problems solution. If this time is too long, the dialogue with the DM, even very convenient, may not occur (in case the DM loses the patience to wait for the solution of the scalarizing problem).

The interactive methods for solving linear integer problems of multicriteria optimization can be divided in two main groups. The interactive methods from the first group [7, 9, 13] are modifications of single-criterion integer methods (usually methods of "branch and bound" type) with the purpose to include the DM in the computing process, so that on the basis of the information provided by him/her, non-dominated integer solutions are obtained. The main difficulties in these interactive methods are connected with the decrease of the number of computational process interrupts and the number of comparisons the DM has to make. The interactive methods from the second group [1, 5, 6, 11, 15, 18, 19] are modifications of the interactive methods solving linear problems of multicriteria optimization with the purpose to include constraints for integrity on a part or on all the variables. The "time factor" of the scalarizing problem solution is accounted to a different degree in these interactive methods. For this purpose the number of the integer problems solved is decreased, approximate algorithms are applied to solve the integer problems or possibility to interrupt the exact algorithms of these problems solution is provided; continuous problems are solved instead of integer problems (especially in the process of DM's learning) and the (weak) non-dominated solutions obtained are presented for evaluation to the DM, etc. Some of the interactive methods operate with aspiration levels of the criteria, others use weighing coefficients for the relative importance of the criteria. The majority of the methods offer one (weak) non-dominated solution at every iteration, the rest – several (weak) non-dominated solutions for evaluation to the DM. The application of the classification-oriented scalarizing problems is more frequently met in the linear integer interactive methods than in the linear interactive methods. Their use in them, anyway, is connected with the possibilities to decrease the waiting time for new solutions evaluated by the DM rather than with expansion of the information, with the help of which the DM may describe his/her local preferences.

A learning-oriented interactive method is suggested in the present paper on the basis of classification-oriented scalarizing problems, that to a high degree includes the positive aspects of the interactive methods for solving linear and linear integer problems of multicriteria optimization, developed up to now. The main features of this interactive method are as follows:

• A possibility to expand the information, with the help of which the DM may set his/her local preferences; besides the desired and acceptable levels of the criteria, he can also set desired and acceptable directions and intervals of alteration in the values of the criteria;

• A possibility to obtain continuous solutions and also approximate integer solutions, thus decreasing the waiting time for the DM in solving linear integer multicriteria problems;

• A decrease in the number of the integer scalarizing problems being solved;

• A possibility for comparatively rapid DM's learning with respect to the multicriteria problems being solved, providing at each iteration continuous, (weak) nondominated solutions or more near (weak) non-dominated solutions for evaluation, as well as free movement of the DM across the whole region of these solutions;

• Comparatively easy evaluation of the solutions presented to the DM, due to the fact that they are to a great extent close one to another.

2. Problem formulation

The linear mixed integer problem of multicriteria optimization (denoted by I), can be formulated as follows:

(1) $$\text{"max"} \ \{f_k(x)\},\ k \in K,$$

subject to:

(2) $$\sum_{j\in N} a_{ij} x_j \le b_i,\ \ i \in M,$$

(3) , $j \in N$,

(4) x_j – integers, $j \in N'$; $N' \subseteq N$,

where , are linear criteria (objective functions),

"max" denotes that all the objective functions should be simultaneously maximized; is the variables vector; $K = \{1, 2, ..., p\}$, $M = \{1, 2, ..., m\}$, $N = \{1, 2, ..., n\}$ and $N' \subseteq N$ are the sets of the indices of the linear criteria (objective functions), of the linear constraints, of the variables and of the integer variables respectively. Constraints (2)-(4) define the feasible region X_1 for the variables of the mixed integer problem. Problem (1)-(3) is a linear problem of multicriteria optimization (P), which is a relaxation of (I). The feasible region for the variables of the linear problem is denoted by X_2.

Several definitions will be introduced for greater precision.

Definition 1. The solution x is called an efficient solution of problem (I) or (P), if there does not exist any other $\bar{x}$ solution, so that the following inequalities are satisfied:

$$f_k(\bar{x}) \geq f_k(x) \text{ for every } k \in K \text{ and}$$
$$f_k(\bar{x}) > f_k(x) \text{ for at least one index } k \in K.$$

Definition 2. The solution x is called a weak efficient solution of problem (I) or (P), if there does not exist another solution such that the following inequalities hold:

$$f_k(\bar{x}) > f_k(x) \text{ for every } k \in K.$$

Definition 3. The solution x is called a (weak) efficient solution, if x is either an efficient or a weak efficient solution.

Definition 4. The vector is called a (weak) non-dominated solution in the criteria space, if x is a (weak) efficient solution in the variable space.

Definition 5. A near (weak) non-dominated solution is a feasible solution in the criteria space, located comparatively near to the (weak) non-dominated solutions.

Definition 6. A current preferred solution is a (weak) non-dominated solution or near (weak) non-dominated solution, if selected by the DM at the current iteration.

Definition 7. The most preferred solution is the current preferred solution, which satisfies the DM to the highest degree.

3. Classification-based scalarizing problems

The type of the scalarizing problem used lies in the basis of each interactive method. The scalarizing problem is a problem of single-criterion optimization and its optimal solution is a (weak) non-dominated solution of the multicriteria optimization problem. The clasification-based scalarizing problems are particularly appropriate in solving linear mixed integer multicriteria optimization problems, because they lead to the decrease of the computing difficulties connected with their solution as well as to the increase of DM's possibilities in describing his/her local preferences and also to reduction of the requirements towards the DM in the comparison and evaluation of the new solutions obtained. In the scalarizing problems used the DM can present his/her local preferences in terms of desired or acceptable levels and also of directions and intervals of alteration in the values of separate criteria. Depending on these preferences, the criteria set can be divided into seven or fewer criteria classes $K^{>}$, $K^{\geq}$, $K^{=}$, $K^{<}$, $K^{\leq}$, $K^{><}$ and K^{0}. The criterion $f_k(x)$, $k \in K$, may belong to one and only one of the following classes:

$k \in K^{>}$, if the DM wishes the criterion $f_k(x)$ to be improved;

$k \in K^{\geq}$, if the DM wishes the criterion $f_k(x)$ to be improved by any desired (aspiration) value Δ_k;

$k \in K^{<}$, if the DM agrees the criterion $f_k(x)$ to be worsened;

$k \in K^{\leq}$, in case the DM agrees the value of the criterion $f_k(x)$ to be deteriorated by no more than δ_k;

$k \in K^{><}$, if the DM wishes the value of the criterion $f_k(x)$ to be in definite limits with respect to the current value f_k , $(f_k - t_k^{-} < f_k(x) \leq f_k + t_k^{+})$;

$k \in K^{=}$, if the current value of the criterion $f_k(x)$ is acceptable for the DM;

$k \in K^{0}$, if the DM is not interested at the moment in the alteration of the criterion $f_k(x)$ and this criterion can be freely altered.

In order to obtain a solution, which is better than the current (weak) non-dominated solution of the linear or linear mixed integer problem of multicriteria optimization, the following Chebyshev's scalarizing problems can be applied on the basis of the implicit criteria classification done by the DM. The first mixed integer scalarizing problem [20], called **DAL1** (desired or acceptable level) uses the sets $K^{\geq}$, $K^{=}$, $K^{\leq}$ and K^{0} and has the following type:

To minimize:

(5)

under constraints:

(6) $$f_k(x) \geq f_k, \quad k \in K^{=},$$

(7) $$f_k(x) \geq f_k - \delta_k, \quad k \in K^{\leq},$$

(8) $$x \in X_1,$$

where:

f_k, $k \in K$, is the value of the criterion $f_k(x)$ in the current preferred solution;

$\bar{f}_k = f_k + \Delta_k$, $k \in K^{\geq}$, is the desired level of the criterion $f_k(x)$;

$f_k'(x)$, $k \in K$, is a scaling coefficient:

where ε is a small positive number.

Since the objective function of the scalarizing problem **DAL1** is a nonlinear function, one may solve the following equivalent linear mixed integer programming problem **DAL11**:

(9) $\min \alpha$

under the constraints:

(10)

(11)

and the constraints (6)-(8).

(12) α – arbitrary.

The values of the objective functions of **DAL-1** and **DAL-11** problems for their optimal solutions are equal [20].

The scalarizing problem **DAL-11** has three properties that enable to a large extent the overcoming of the computing difficulties connected with its solution as an integer programming problem and also the decrease of DM's efforts in the comparison of new solutions. The first property is connected with this, that the current preferred integer solution (found at the previous iteration) of the multicriteria problem is a feasible solution of **DAL-11** problem. This facilitates the exact as well as the approximate algorithms for solving **DAL-11** problem, because they start with a known initial feasible integer solution. The second property is connected with the fact that the feasible region of **DAL-11** problem is a part of the feasible region of the multicriteria problem (I), differing from the feasible regions of the reference point scalarizing problems, which coincide with the feasible region of problem (I). Depending on the values of the parameters Δ_k, $k \in K^{\geq}$, δ_k, $k \in K^{\leq}$, the feasible region of problem **DAL-11** can be comparatively narrow and the feasible solutions in the criteria space, found with the help of exact or approximate algorithms of integer programming, may be positioned very close to the non-dominated surface of the multicriteria problem (I). The obtaining and use of such approximate (weak) non-dominated solutions may decrease considerably the time, during which the DM waits for new solutions evaluation. Insignificant decrease in the quality of the solutions obtained can significantly improve the dialogue with the DM. The third property is connected with DM's possibility to realize a search strategy of "not big profits-small losses" type. This is due to the fact that with the help of **DAL-11** an optimal solution is sought, which minimizes Chebyshev's distance from the feasible criteria defined up to the current reference point, the components of which are equal to the desired by the DM values of the criteria being improved and the current values of the criteria being deteriorated. The (weak) non-dominated solution obtained and the current solution are relatively close and the DM can make his choice easier. This is also in power when the scalarizing problem is solved approximately and more feasible solutions are obtained, that are comparatively near to the current solution and between also. In other words, the influence of the so-called “limited comparability” of the (weak) non-dominated solutions is diminished.

The classification-oriented scalarizing problem **DAL11** is particularly appropriate in solving integer problems of multicriteria optimization, because they can reduce the computing difficulties, connected with their solving, and also decrease the requirements towards the DM in the comparison and evaluation of the new solutions obtained. From a viewpoint of the information, demanded from the DM in the search for new solutions, these scalarizing problems are comparatively close to scalarizing problems of the reference point [22], but unlike them, here the DM is not obliged to define the desired and acceptable levels of all the criteria. In Chebyshev’s scalarizing problem called **DALDI-1** (desired or acceptable level, direction and interval) below presented, the DM can present his/her local preferences not only by desired and acceptable levels, but by desired and acceptable directions and intervals of change in the values of the separate criteria as well.

The mixed integer scalarizing problem **DALDI-1** [21] uses the sets $K^{>}$, $K^{\geq}$, $K^{=}$, $K^{<}$, $K^{\leq}$, $K^{><}$ and K^{0} and has the following type:

to minimize

(13)

under the constraints:

(14)

(15)

(16)

(17)

(18) $$x \in X_1,$$

where:

f_k, $k \in K$, is the value of the criterion $f_k(x)$ in the current preferred (weak) non-dominated solution;

$\bar{f}_k = f_k + \Delta_k$, $k \in K^{\geq}$, is the desired level of the criterion;

f'_k, $k \in K$, is a scaling coefficient, defined in the following way:

where ε is a small positive number.

In order to obtain a solution better than the current (weak) non-dominated solution of the linear problem of multicriteria optimization, Chebyshev’s continuous scalarizing problem **DALDI-2** has to be used, which is obtained from the integer scalarizing **DALDI-1** problem, replacing constraint (18) by the following constraint:

(19) $$x \in X_2.$$

Since the objective function (1) of the integer scalarizing problem **DALDI-1** is a nonlinear function, the following equivalent problem of linear mixed integer programming **DALDI-11** can be solved instead:

(20) $$\min (\alpha + \beta)$$

under constraints:

(21)

(22)

(23)

(24) $$\alpha, \beta \text{ – arbitrary,}$$

and constraints (14)-(18)

The values of the objective functions of problems **DALDI-1** and **DALDI-11** for their optimal solutions are equal [21].

Instead of a continuous scalarizing problem **DALDI-2**, the objective function of which is also non-differentiable, a problem of linear programming **DALDI-21**, equivalent to it, may be solved. It is obtained from **DALDI-11** problem, replacing constraint (18) by constraint:

(25) $$x \in X_2.$$

Let us assume that a (weak) non-dominated solution of the linear problem of multicriteria optimization is found solving scalarizing problem **DALDI-21** and that we wish to find a (weak) non-dominated solution of the multicriteria optimization located close to the (weak) non-dominated solution of the linear problem of multicriteria optimization. Let us denote the (weak) non-dominated solution of the linear problem of multicriteria optimization by . To find a (weak) non-dominated solution of the linear integer problem of multicriteria optimization, positioned near to a (weak) non-dominated solution of the linear problem of multicriteria optimization, the following problem of mixed integer programming **DALDI-31** [22], may be applied

(26) $$\min \alpha$$

under the constraints:

(27) $$k \in K,$$

(28) $$x \in X_1,$$

(29) $$\alpha \text{ – arbitrary,}$$

where:

ε is a small positive number.

The scalarizing problem **DALDI-11** has characteristics similar to **DAL-11** scalarizing problem. Still there are two differences between them. The first difference consists in this, that **DALDI-11** scalarizing problem gives greater freedom to the DM when expressing his/her local preferences in the search for a better (weak) non-dominated solution. Besides desired and acceptable values of a part or of all the criteria, the DM has the possibility to set also desired or acceptable directions and intervals of change in the values of some criteria. In this way the DM can describe his/her local preferences with greater flexibility, accuracy and reliability. The second difference between **DAL-11** and **DALDI-11** scalarizing problems concerns the possibility to alter their feasible sets (make them "narrower"), so that their feasible solutions are positioned close to the non-dominated (efficient) solutions of the multicriteria problem. The more the criteria are, which the DM wishes to be freely improved or freely deteriorated, the smaller this possibility is. The narrow feasible regions of scalarizing problems **DAL-11** and **DALDI-11** enable the successful application of approximate single-criterion algorithms, which is especially important when these problems are integer. It can be easily observed that scalarizing problem **DAL-11** is better than scalarizing problem **DALDI-11** in this aspect .

4. GAMMA-I2 interactive method

On the basis of scalarizing problems **DAL-11, DALDI-11, DALDI-21** and **DALDI-31**, a classification-oriented interactive method, called **GAMMA-I2** is suggested for solving linear integer programming problems of multicriteria optimization. Scalarizing problems **DAL-11, DALDI-11** and **DALDI-31** are mixed integer programming problems. The problems of mixed integer programming are NP-problems, i.e. the time for their exact solution is an exponential function of their dimension. That is why, in solving integer problems, particularly problems of larger dimension (above 100 variables and constraints), some approximate methods are used [4, 12, 14, 17]. Since finding a feasible solution is as difficult as finding an optimal solution, in the general case the approximate integer methods do not guarantee the finding of neither an optimal integer solution, nor an initial feasible integer solution. Nevertheless, if the initial feasible integer solution is known and the feasible region is comparatively "narrow", then with the help of the approximate integer methods, especially approximate integer methods, realizing some of the known meta-heuristics like "tabu search" [4], "simulated hardening" [12] and "evolutionary search" [14], some satisfactory and in many cases - optimal integer solutions could be found. The scalarizing problems **DAL-11** and **DALDI-11** have known feasible initial integer solutions.

The scalarizing problems **DALDI-11** and **DALDI-21** allow enlargement of the information, with the help of which the DM can set his/her local preferences, defining besides desired and acceptable criteria levels (as in **DAL-11** problem), also desired and acceptable directions and intervals of change in the criteria values. But this expansion of the information defined by the DM in **DALDI-11** problem, leads to extension of the feasible set of criteria alteration in the criteria space and of the integer variables in the variables space. Hence, the approximate integer solutions of **DAL-11** problem (obtained with the help of an approximate integer method) are situated closer to the non-dominated (efficient) surface of the multicriteria problem, than the approximate solutions of **DALDI-11** problems. In this connection, when solving multicriteria problems of (I) type with large dimension, when the scalarizing problems must be approximately solved in order to reduce the waiting time for new solutions evaluated by the DM, it is better to use **DAL-11** scalarizing problem than scalarizing problem **DALDI-11**.

The interactive **GAMMA-I2** method proposed is a method oriented towards learning. This means that the DM can seek freely the final or the most preferred solution of the output multicriteria problem from the sets of the (weak) non-dominated or approximate (weak) non-dominated solutions. For this purpose, the DM has to acquire in the learning phase any idea about these sets, about the feasible ranges of criteria alteration, about some general relations between the alterations of the separate criteria. In connection with this, besides including of definite scalarizing problems **DAL-11, DALDI-11, DALDI-21** and **DALDI-31**, three different strategies of search for new solutions subjected to evaluation, are also applied in the development of **GAMMA-I2** interactive method. The first strategy, called integer strategy, is a search at each iteration of a non-dominated integer or (weak) non-dominated integer solution, solving exactly the corresponding integer scalarizing problems. The second strategy, called approximate integer strategy, is seeking approximate (weak) non-dominated integer solutions at some iterations, approximately solving the respective integer scalarizing problems. During the learning phase, and in problems of large dimension up to the very end, only approximate (weak) non-dominated solutions may be sought. The third strategy, called mixed strategy, is search for continuous (weak) non-dominated solutions at most of the iterations, solving continuous scalarizing problems, at that with respect to the current continuous (weak) non-dominated solution found, a close to it (weak) non-dominated integer or approximate (weak) non-dominated integer solution is sought from time to time. The search for continuous (weak) non-dominated solutions consists in solving the continuous problem of multicriteria optimization (P). As a (weak) non-dominated integer solution or an approximate (weak) non-dominated integer solution that solution is accepted in the criteria space of problem (I), the solution of which in the variables space is an integer solution. .

The first searching strategy is appropriate for solving integer problems of multicriteria optimization of small and average dimension. Then scalarizing problem **DALDI-11** is applied. The second seeking strategy is necessary in solving problems of large dimension (above 50 variables and constraints). In this case the use of scalarizing problem **DAL-11** is recommended. On the account of DM's possibilities reduction in describing his/her local preferences, the quality of the approximate solutions obtained can be improved. The third seeking strategy is also appropriate in solving problems of large dimension. Scalarizing problems **DALDI-11, DALDI-21** and **DALDI-31** may be applied in this strategy.

GAMMA-I2 interactive method is intended to solve integer problems of multicriteria optimization. In order to overcome the computational difficulties (especially when solving problems of large dimension), the three strategies above described are realized in the method for seeking new solutions of evaluation. The method is oriented towards learning and the DM has to determine when the most preferred solution is found.

The algorithmic scheme of the interactive method **GAMMA-I2** consists in the following main steps:

Step 1. An initial continuous non-dominated solution is found, setting $f_k = 1$, $k \in K$, and $\bar{f}_k = 2$, $k \in K$, and solving **DALDI-21** problem.

Step 2. A question is set to the DM what type of a new solution has to be searched for – a continuous or an integer one. In case an integer solution is sought, Step 6 is executed, otherwise – Step 3.

Step 3. The DM is asked to define the desired and acceptable levels, directions and intervals of change for a part or for all the criteria.

Step 4. **DALDI-21** scalarizing problem is solved. The continuous (weak) non-dominated solution obtained is presented to the DM for evaluation. In case the DM wishes to see an integer solution, located near to the continuous solution obtained, Step 5 is executed, otherwise – Step 2.

Step 5. **DALDI-31** problem is solved and the integer solution obtained is presented for evaluation to the DM. If the DM considers this solution as the most preferred solution of the output multicriteria problem, Step 10 is executed, otherwise – Step 2.

Step 6. A question towards the DM what integer solution he prefers to see – a (weak) non-dominated or an approximate (weak) non-dominated solution. In the first two cases the algorithm passes to Step 8, in the last one – to Step 7.

Step 7. The DM is requested to define the desired and acceptable levels of the values for a part or for all the criteria. Approximate solution of **DAL-11** problem. The set of the approximate (weak) non-dominated solutions found are presented to the DM for evaluation and selection. In case the DM evaluates and chooses one of these solutions as the most preferred solution of the output multicriteria problem, Step 9 is executed, otherwise – Step 2.

Step 8. A requirement to the DM to define the desired and acceptable levels, directions and intervals of alteration in the values of a part or of all the criteria. **DALDI-11** scalarizing problem is solved. The integer solution obtained is presented to the DM for evaluation. In case the DM estimates this solution as the most preferred solution of the output multicriteria problem, Step 9 is executed, otherwise – Step 2.

Step 9. Stopping the process of solving the linear integer multicriteria problem.

5. Concluding remarks

A learning-oriented interactive method **GAMMA-I2** for solving linear mixed integer problems of multicriteria programming is proposed. The method offers the DM at each iteration the flexibility to express his/her preferences with respect to the current preferred solution and the possibility to select for computing one or more (weak) nondominated (continuous or integer) solutions or near (weak) nondominated integer solutions. In the learning phase or when solving large problems, the DM can solve continuous scalarizing problems or integer scalarizing problems approximately at many iterations. This considerably reduces the computational time at each iteration.

The methods **GAMMA-I2** together with **GAMMA-L** method (V a s s i l e v et a l.) and **GAMMA-I1** method (V a s s i l e v e t a l., [18]) are realized in an experimental software system **MOLIP**, developed at the Institute of Information Technologies of the Bulgarian Academy of Sciences. This system is designed for interactive solving of linear and linear mixed integer problems of multicriteria optimization.

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Метод, ориентиран към обучение, предназначен за линейна смесена целочислена многокритериална оптимизация

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(Р е з ю м е)

В статията е описан метод, ориентиран към обучение, за решаване на линейни смесени целочислени задачи на многокритериалната оптимизация. Методът предлага на лицето вземащо решение (ЛВР) гъвкавост при изразяване на предпочитанията му. ЛВР се поощрява, особено във фазата на обучение или когато се решават големи задачи, да решава непрекъснати скаларизиращи задачи или целочислени скаларизиращи задачи приближено на много итерации. Методът е реализиран в експериментална софтуерна система, подпомагаща решаването на линейни смесени целочислени многокритериални оптимизационни задачи.