

## Predictive Functional Control Using a Blending Approach

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**Abstract:** *A new strategy for controlling nonlinear parametrically dependent plants is proposed in order to ensure a good control performance and keep up the simplicity of Predictive Functional Control (PFC). Two alternative approaches are considered to realize a quasi linear PFC: manipulated variable blending and neighboring models blending. In this way it is avoided a nonlinear (NP) problem solving at each sampling time instant. Very simple no iterative calculations are needed. This implies that the computational load is slightly heavier in comparison with linear PFC (LPFC). The modified quasi-linear PFC algorithms are applied to a pH waste water neutralization laboratory stand. A comparative analysis is carried out between the proposed blending alternative approaches. Their performance is also compared with PFC and PID control schemes.*

**Keywords:** *Predictive Functional Control, blending, pH neutralization, nonlinear parametrically dependent plant.*

### I. Introduction

Model predictive control (MPC) is established approach in theoretical investigations and industry with a stable increase of popularity in the last decade [3, 13]. A few companies have obtained commercial success companies – ASPEN TECH, HONEYWELL ( HI Spec), ADERSA, SHELL – Global Solutions. The total estimation shows that linear MPC (LMPC) applications are more than 4500 worldwide [12, 14] . The theoretical achievements in LMPC and increased number of computations result in appearing of fourth generation commercial codes – DMC +

(ASPEN Tech) and RMPCT (HONEYWELL) which possess some important new peculiarities – improved identification technology, robust control design, steady-state optimization, new humane-machine interfaces [12]. At the same time nonlinear MPC (NMPC) still has a limited marketplace [1] much smaller than that of linear MPC (LMPC). The reason is in the computational complexity of NMPC algorithms.

Both academic researches and industrial development in NMPC area continue to grow. The main efforts are directed to obtain practical control calculations.

A successive linearization procedure and extended Kalman filter (EKF) are used at each sample time instant to solve a quadratic programming problem (QP) [10]. The prediction equation can be computed via no iterative nonlinear integration. A large number of additional tuning parameters from EKF may complicate online tuning procedures since the performance and stability of the overall closed loop dependent also upon state estimation [16]. In [3] an algorithm is proposed to determine the solution of robust optimal control as a function of the initial state.

The on-line computation of the resulting MPC controller is reduced to a simpler linear function evaluation. Two new NMPC techniques and special dynamic optimizers that require reduced computational load are proposed in [5] but the nonlinear optimization problem has to be solved at each sampling instance. In [11] an improvement is proposed in solving nonlinear programming problem for each time step and for each search step in order to guarantee convexity of the objective function in a constrained NMPC algorithm. The difficulties with the efficient solution of nonconvex optimization problem of NMPC on-line are referenced in [15]. By applying a newton-type algorithm which exploits the structure of NMPC problems authors achieve a very few iterations compared to general purpose nonlinear programming solvers. In [16] two stage procedure in NMPC is accepted: firstly the optimal linear unconstrained input sequence is obtained by solving algebraic Riccati equation, second, by using a local linearization of the nonlinear model around a predetermined target trajectory a quadratic programming problem (QP) is solved at each time step. A Winer model based NMPC is considered in [6]. The moderate level of computational complexity is retained by combination of LMPC and gain scheduling.

In this paper a Predictive Functional Control strategy [13, 14] is proposed to control strongly nonlinear plants. The developed procedure does not solve nonlinear programming problem at each time period in order to calculate control action within a fixed sampling time instant. A Multi-model approach is accepted to describe the parametrically dependent plant model used by the procedure. Local time invariant models are defined for a number of points of the static process characteristic Fig. 4. Two effective blending approaches are studied – blending of manipulated variables (Output Blending Control Scheme) calculated for two neighboring models, quasi LPFC based on a model with blended coefficients (Model Blending Control Scheme). Some preliminary investigations in nonlinear parametrically dependent plants are reported in [2, 18].

In this paper a comparative analysis of both blending approaches is carried out. The developed procedures are also compared to LPFC and PID control schemes. The proposed approach differs from both parametrical PFC [13] and gain scheduling control [7, 8, 17]. Simulation experiments are fulfilled with pH neutralization waste water stand.

## II. Basic predictive functional control algorithm

The basic Predictive Functional Control Algorithm derived by J.Richalet is described in details in [13]. The main concept of the algorithm is presented in Fig. 1.

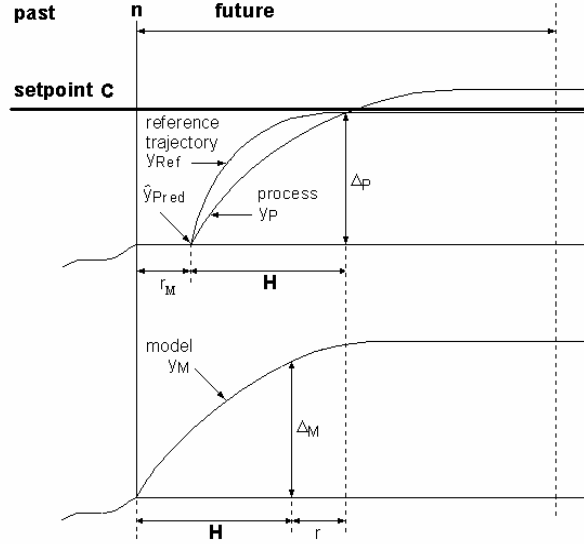


Fig. 1. PFC basic concepts

An exponential reference trajectory  $y_{\text{Ref}}$  with a time constant  $T_R$  is defined and initialized on either the measured or predicted process output  $y_{\text{Pred}}$  depending on model time delay presence. We look for a control action which assures intersection between future process output  $y_p$  and the reference trajectory at the coincidence  $H$ . The structure of the manipulated variable must be a priori considered as a combination of basis functions. In this approach step-like basis functions are accepted. We assume that the process is described by first order lag with pure time delay:

$$(1) \quad W(p) = \frac{k}{Tp + 1} e^{-pd}.$$

By sampling the continuous transfer functions (1), the process model can be obtained in the following form :

$$y_M(n+1) = \alpha_M y_M(n) + k_M(1 - \alpha_M)u(n - r_M),$$

where  $T_0$  is sample time,  $\alpha_M = e^{-T_0/T}$ ,  $r_M = d/T_0$ ,  $k_M = k$ .

PFC algorithm uses a special technique to control delayed plants with non-delayed models that simplifies the computations and finally the process model becomes:

$$y_M(n+1) = \alpha_M y_M(n) + k_M(1 - \alpha_M)u(n).$$

Considering Fig. 1, the following equations can be written:

$$(2) \quad C(n+r+H) - y_{\text{Ref}}(n+r+H) = \lambda^H (C(n+r) - \hat{y}_{\text{Pred}}),$$

$$(3) \quad y_{\text{Ref}}(n+r+H) = y_p(n+r+H),$$

where  $\lambda = e^{-\frac{T_0}{T_R}}$ . From (2) and (3) we can obtain:

$$(4) \quad y_P(n+r+H) = C(n+r+H) - \lambda^H (C(n+r) - \hat{y}_{\text{Pred}}).$$

The future model increment  $A_p$  can be expressed by the following equation:

$$(5) \quad \Delta_p = y_P(n+r+H) - \hat{y}_{\text{Pred}}.$$

Let us assume that the future setpoint is constant throughout the whole prediction horizon, i.e.

$$(6) \quad C(n+r+H) = C(n+r) = C = \text{const.}$$

The predicted value of the process  $\hat{f}_{\text{Pred}}$  is given by the following equation since the process model is non-delayed:

$$(7) \quad \hat{y}_{\text{Pred}} = y_P(n) - y_M(n-r) + y_M(n).$$

The PFC approach states that the desired increment of the process should be equal to the model increment  $\Delta_p = \Delta_M$ , thus:

$$(8) \quad \Delta_p = y_M(n+H) - y_M(n).$$

If we combine (4), (5), (6) and (8), the following equation can be written

$$(9) \quad (1 - \lambda^H)(C - \hat{y}_{\text{Pred}}) = y_M(n+H) - y_M(n).$$

The application of step-like basis functions gives the following expressions for the future model output at the  $H$ -th time instant:

$$(10) \quad y_M(n+H) = \alpha_m^H y_M(n) + K_m(1 - \alpha_m^H)u(n).$$

We can write the following expression for the control action with respect to (9) and (10):

$$(11) \quad u(n) = \frac{(1 - \lambda^H)(C - \hat{y}_{\text{Pred}})}{K_m(1 - \alpha_m^H)} + \frac{y_M(n)}{K_m}.$$

### III. Problem statement

In industry there is a variety of plants in which the model coefficients depend on parameters that are functions of time or other parameters. Some of these typical plants are heat exchangers, in which the parametric variable is the load; reheating furnaces where the parametric variable is the current capacity; gas absorbers in which the absorbent is a parametric variable etc. Two different types of influences act on these plants - co-ordinate, which acts on the output variable directly and parametric, which acts on the plant parameters. The transfer function this type of plants can be written in the following form:

$$(12) \quad W(p, \psi) = \frac{k(\psi)}{T(\psi)p + 1} e^{-pd(\psi)},$$

where  $\psi$  can be one of the following variables – process input, process output, external parameter. These plants are well studied and some control schemes are proposed in [8, 17]. In this paper a pH neutralization stand is taken under consideration. As can be seen (Fig. 4) the pH plant has extremely nonlinear static characteristic. In this case the plant output (potential of hydrogen (pH)) plays the role of the parametric variable. For each “critical” point on the static characteristic a local linear time invariant model is identified. “Critical” points are defined as local extrema of static characteristic derivative function.

#### IV. Output blending control scheme for nonlinear plants

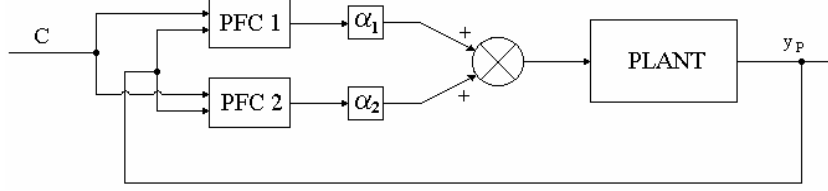


Fig. 2. Output blending control scheme

The output blending control scheme with several parallel working Predictive Functional controllers (Fig. 2), is proposed by H a d j i s k i and A s e n o v, [2]. The main idea of the control strategy is implementation of several simple PFCs. At each time instant the control action is represented as a weighted sum of two neighboring controllers' outputs. The weight coefficients are proportional to the "distance" of the parametric variable to the local models (13)

$$(13) \quad \alpha_1 = \frac{\beta - \beta_1}{\beta_2 - \beta_1}, \quad \alpha_2 = \frac{\beta_2 - \beta}{\beta_2 - \beta_1} = 1 - \alpha_1,$$

where  $\beta$  is the current value of the parametric variable  $\beta_1 \leq \beta \leq \beta_2$ ;  
 $\beta_1, \beta_2$  – the values of the parametric variable for which the local models are identified.

Then the blended output can be computed by the next expression:

$$(14) \quad U(n) = \alpha_1 U_1(n) + \alpha_2 U_2(n),$$

where  $U_1(n)$  and  $U_2(n)$  are the local controllers outputs. At each time instant as the setpoint moves to different points the control action is calculated through (14) but different neighboring models are used always indexed with 1 and 2.

#### V. Model blending control scheme for nonlinear plants

A similar control strategy is considered in [18]. The basic concept is to compute the control action on the basis of an equivalent linear plant model:

$$(15) \quad W_{\text{eq}}(p) = \frac{k_{\text{eq}}}{T_{\text{eq}}p + 1} e^{-pd_{\text{eq}}}.$$

The equivalent linear model coefficients can be computed proportionally to the distance of the local models:

$$(16) \quad \begin{aligned} k_{\text{eq}} &= \alpha_1 k_1 + \alpha_2 k_2, \\ T_{\text{eq}} &= \alpha_1 T_1 + \alpha_2 T_2, \\ d_{\text{eq}} &= \alpha_1 d_1 + \alpha_2 d_2, \\ y_M(n) &= \alpha_1 y_{M1}(n) + \alpha_2 y_{M2}(n). \end{aligned}$$

The control action is computed through formula (11) where  $K_m, \alpha_m$  and  $y_{\text{Pred}}$  are obtained from the equivalent model.

## VI. Output blending and model blending control schemes comparison

Let us take the simplest case where the nonlinear plant is represented by a Winner model connected linear dynamical part and nonlinear static part in series. We can rewrite (1) and (11) in the following  $z$  form

$$(17) \quad u(z) = \frac{(1 - \lambda^H)(C - y_P(z))}{K_m(1 - \alpha_m^H)} + \frac{y_M(z)}{K_m},$$

$$(18) \quad y_M(z) = \frac{K_m(1 - \alpha_m)}{1 - \alpha_m z^{-1}} z^{-1},$$

which combination gives

$$(19) \quad u(z) = \frac{e(z)(1 - \lambda^H)(1 - \alpha_m z^{-1})}{K_{m2}(1 - \alpha_m^H)(1 - z^{-1})},$$

where  $e(z) = C - y_P(z)$ .

• **Output blending approach.** Since we have assumed that the process model is represented by a Winner model, all local models with some degree of approximation would have the same dynamic part but different gains. Thus for the local neighboring controllers' outputs the following expressions can be written:

$$(20) \quad u_1(z) = \frac{e(z)(1 - \lambda^H)(1 - \alpha_m z^{-1})}{K_{m1}(1 - \alpha_m^H)(1 - z^{-1})},$$

$$(21) \quad u_2(z) = \frac{e(z)(1 - \lambda^H)(1 - \alpha_m z^{-1})}{K_{m2}(1 - \alpha_m^H)(1 - z^{-1})}.$$

Considering (14), (20) and (21) we can write the final formula for the control action produced by output blending control scheme

$$(22) \quad u_{\text{outbl}}(z) = \frac{e(z)(1 - \lambda^H)(1 - \alpha_m z^{-1})}{(1 - \alpha_m^H)(1 - z^{-1})} \left( \frac{\alpha_1}{K_{m1}} + \frac{\alpha_2}{K_{m2}} \right).$$

Comparing (22) and (11), the following expression for the "equivalent" model gain can be written:

$$(23) \quad K_{\text{outbl}} = \frac{K_{m1}K_{m2}}{\alpha_1 K_{m2} + \alpha_2 K_{m1}}.$$

• **Model Blending Approach.** Considering (16) and the assumption made above the following equations for the process model and the control action can be written:

$$(24) \quad y_M(z) = \alpha_1 y_{M1}(z) + \alpha_2 y_{M2}(z),$$

$$(25) \quad y_M(z) = \frac{K_{eq}(1 - \alpha_m)}{1 - \alpha_m z^{-1}} z^{-1},$$

$$(26) \quad u_{\text{modbl}}(z) = \frac{e(z)(1 - \lambda^H)(1 - a_m z^{-1})}{(1 - \alpha_m^H)(1 - z^{-1})} \frac{1}{K_{\text{eq}}}.$$

As it can be seen from equations (22) and (26) the difference between both blending approaches is in the manner in which the “equivalent” coefficients are computed. Although they apply different “equivalent” coefficients both control values are quite close for the considered pH neutralization stand (Figs. 6-9).

## VII. Simulation results

The pH neutralization stand scheme comprises three tanks for different acids – Tank 1, Tank 2 and Tank 3 and one tank for a base – Tank 4 (Fig. 4) [4].

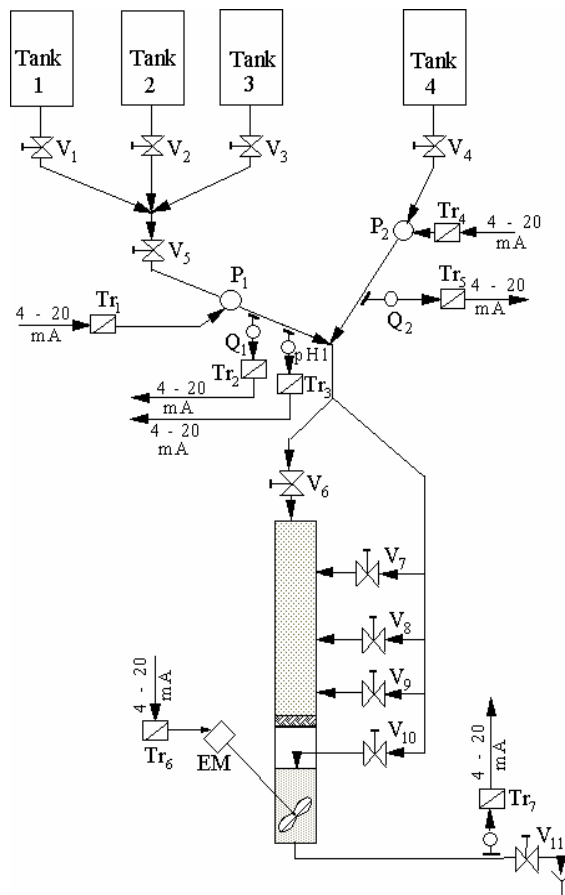


Fig. 3. pH pilot plant

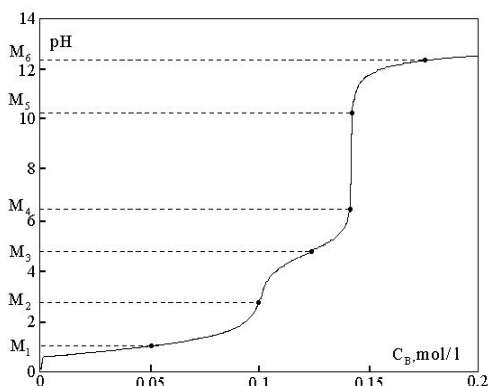


Fig. 4. pH pilot plant static characteristics

The control valves  $V_j-V_5$  ensure the feeding of the reaction part of the installation with desired combination of strong, medium and weak acids as well as with a strong base. This allows to obtain the nonlinear titration curve for given combination of acids in a large regime area. The reaction area of the installation

considers two sections – diffusion section and stirring section. The purpose of the diffusion section is to ensure a desirable transport time delay by using different valves  $V_6-V_{10}$ . The purpose of the stirring section is to mix the reacting components. The used strong acids are: HCl (Tank 1) and  $H_2SO_4$  (Tank 2). The weak acid is  $CH_3COOH$  (Tank 3). The neutralization reagent is sodium hydroxide NaOH, which is a strong base (Tank 4). In Fig. 5 it is presented a titration curve received as a combination of the given acids and a base. As can be seen it is strongly nonlinear and its derivative is not monotonic. The values of the local gains differ more than ten times. Control of this plant in full range of operational conditions by controller with fixed tuning parameters is impossible or system performance will be extremely poor and unsatisfactory. In order to overcome the drawbacks of these control problems the proposed above strategy is implemented. According to Fig. 4, six local models  $M_1-M_6$  are derived by experimental identification under the following assumptions: the gain is determined by the titration curve in the corresponding point because the flow rate is almost constant, the dead time and the time constant are equal for the all local models. In Figs. 5 and 6 it is shown the performance of the developed control schemes subjected to step-like setpoint changes without disturbances at the acid flow  $Q_1$ ;  $A$  – controlled variables (pH);  $B$  – manipulated variables (flow  $Q_2$ ). As it can be seen both considered schemes show similar performance. Additionally in Fig. 6 it is shown the system performance with PID controller. PID tuning parameters are chosen in such a way to guarantee optimal control performance in the whole examined region. It is obvious that the PID controller performance is much poorer than that of MB-PFC and OB-PFC schemes. The same variables are shown in Fig. 7 subjected to disturbance at the acid flow  $Q_1$  while the setpoint is kept constant (pH=6). PID performance is still poor while the MB-PFC and OB-PFC schemes act better in a similar manner. In Fig. 8 there are presented the controlled variables ( $A$ ) and manipulated variables ( $B$ ) of the output blending predictive functional control scheme (OB-PFC), and linear predictive functional control scheme (LPFC) with different models subjected to step-like setpoint changes (pH = 5, pH = 6). The gains of local models for the considered points (pH = 5, pH=6) differ about 5 times. LPFC\* curve is obtained by applying the lower gain model (pH=5). On the contrary, LPFC\*\* curve is obtained by applying the higher gain model (pH=6). The LPFC\* curve becomes unstable at pH=5 while LPFC\*\* performance is extremely slow in both directions.



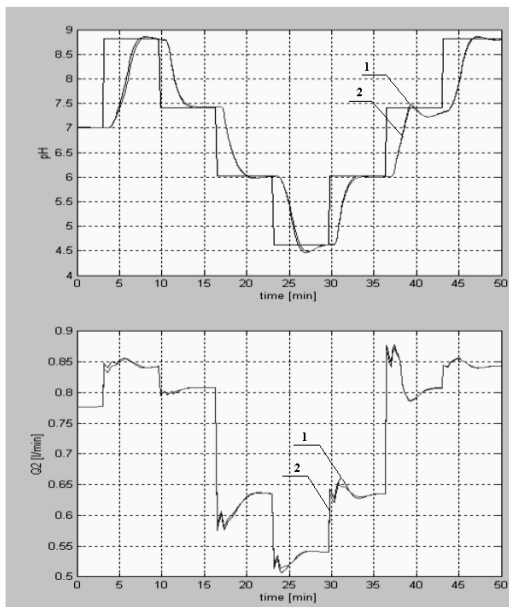


Fig. 5. Model Blending and Output Blending control schemes performance subjected to step-like setpoint changes: 1 – OB-PFC; 2 – MB-PFC

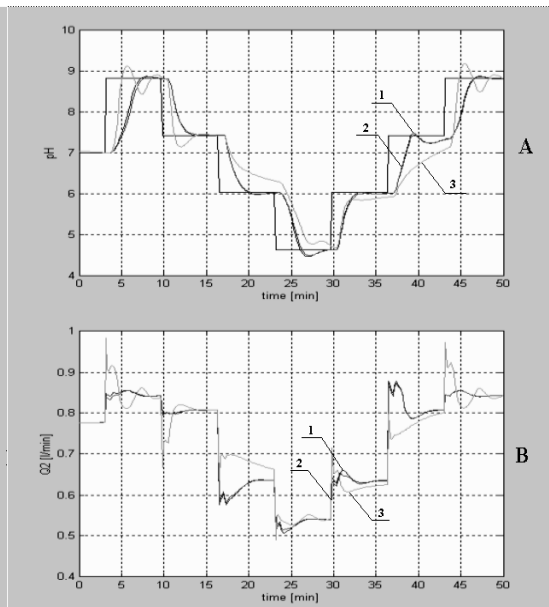


Fig. 6. Model Blending, Output Blending and PID control schemes performance subjected to step-like setpoint changes: 1– OB-PFC; 2 – MB-PFC

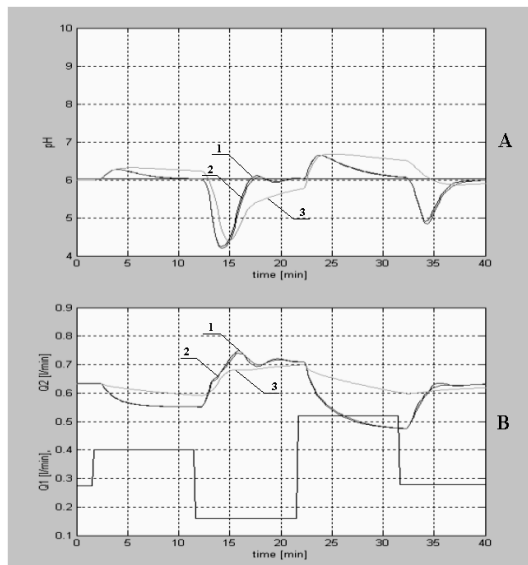


Fig. 7. Model Blending, Output Blending and PID control schemes performance subjected to unmeasured disturbance changes: 1 – OB-PFC; 2 – MB-PFC; 3 – PID

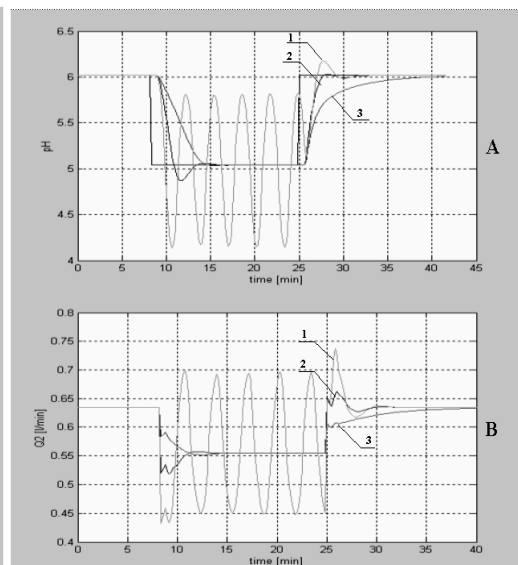


Fig. 8. Linear PFC and Output Blending control schemes performance subjected to setpoint changes: 1 – LPFC\*, 2 – OB-PFC; 3 – LPFC\*\*

## VIII. Conclusions

Two new control strategies (OB-PFC and MB-PFC) for control of nonlinear plants are proposed in order improve the control performance and keep up the simplicity of Predictive Functional control techniques.

The described control algorithm has been applied to a neutralization laboratory stand showing good control performance and similar behavior.

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