

## Grinding in Ball Mills: Modeling and Process Control

*Vladimir Monov, Blagoy Sokolov, Stefan Stoenchev*

*Institute of Information and Communication Technologies, 1113 Sofia*

*E-mails: vmonov@iit.bas.bg blagoysokolov@yahoo.com stoenchev@mail.orbitel.bg*

**Abstract:** *The paper presents an overview of the current methodology and practice in modeling and control of the grinding process in industrial ball mills. Basic kinetic and energy models of the grinding process are described and the most commonly used control strategies are analyzed and discussed.*

**Keywords:** *Ball mills, grinding circuit, process control.*

### I. Introduction

Grinding in ball mills is an important technological process applied to reduce the size of particles which may have different nature and a wide diversity of physical, mechanical and chemical characteristics. Typical examples are the various ores, minerals, limestone, etc. The applications of ball mills are ubiquitous in mineral processing and mining industry, metallurgy, cement production, chemical industry, pharmaceuticals and cosmetics, ceramics, different kinds of laboratory studies and tests. Besides particle size reduction, ball mills are also widely used for mixing, blending and dispersing, amorphisation of materials and mechanical alloying [1, 49, 51].

As a construction, a ball milling device usually consists of a cylindrical vessel mounted on an appropriate basis at both ends which allows rotation of the vessel around the center axis. The mill is driven by a girth gear bolted to the shell of the vessel and a pinion shaft moved by a prime mover. The prime movers are usually synchronous motors equipped with an air clutch or gear transmission. After the mill is charged with the starting material (ore, rock, etc.) and the grinding media (balls),

the milling process takes place during rotation as a result of the transfer of kinetic energy of the moving grinding media into the grinding product.

The design of a ball mill can vary significantly depending on the size, the equipment used to load the starting material (feeders), and the system for discharging the output product. The size of a mill is usually characterized by the ratio “length to diameter” and this ratio most frequently varies from 0.5 to 3.5. The starting material can be loaded either through a spout feeder or by means of a single or double helical scoop feeder. Several types of ball mills are distinguished depending on the discharge system and these types are commonly known as overflow discharge mill, diaphragm or grate discharge mill and centre-periphery discharge mill, e.g. see [23]. In industrial applications, the inner surface of the mill is lined with mill liners protecting the steel body of the mill and incorporating mill lifters which help to raise the content of the mill to greater heights before it drops and cascades down [36].

There are three types of grinding media that are commonly used in ball mills:

- steel and other metal balls;
- metal cylindrical bodies called cylpebs;
- ceramic balls with regular or high density.

Steel and other metal balls are the most frequently used grinding media with sizes of the balls ranging from 10 to 150 mm in diameter [30]. Cylpebs are slightly tapered cylindrical grinding media with rounded edges and equal length and diameter with sizes varying from 8×8 to 45×45 mm. Their shape is developed to maximize the grinding efficiency due to their high density and specific surface area [17]. Ceramic balls with regular density are usually porcelain balls and the high density balls are made with a high alumina oxide content and they are more abrasion resistant. The basic properties of the milling bodies are their mass and size, wear rate, influence on the particle breakage rate and energy efficiency of the grinding process [15, 18, 22, 28]. A comparison between ball mills and cylpebs is made in [42].

The speed of rotation of the mill determines three basic types of operation modes: slow rotation (cascading), fast rotation (cataracting) and very fast rotation (centrifugation). Each type is characterized by a specific trajectory of motion of the charge in the mill and a different impact of the milling bodies on the ground material. The grinding process can also take place in dry or wet conditions depending on whether wetting agents are added to the starting material. Some important characteristics of dry and wet grinding are studied in [29]. The particle size reduction depends on the following basic factors:

- characteristics of the material charged in the mill (mass, volume, hardness, density and size distribution of the charge);
- characteristics of the grinding media (mass, density, ball size distribution);
- speed of rotation of the mill;
- slurry density in case of wet grinding operation.

Quantitative estimations of these parameters can be found in [4, 5, 23].

An important characteristic of an industrial ball mill is its production capacity which is measured in tons of production per hour. The production capacity depends

on mill dimensions, the type of the mill (overflow or grate discharge), the speed of rotation, the mill loading, the final product size required from a given feed size (coefficient of reduction), the work index of the material, the mill shaft power and the specific gravity of the material. These parameters are thoroughly studied in [4, 5] and an empirical relation is suggested expressing the mill capacity as a ratio of the mill shaft power and the energy consumed in the grinding process.

In order to achieve the desired particle size, the milling under industrial conditions is usually performed in grinding circuits including classifiers that separate the material according to particle sizes. The simplest cases of open- and closed-circuit systems are shown in Fig. 1.

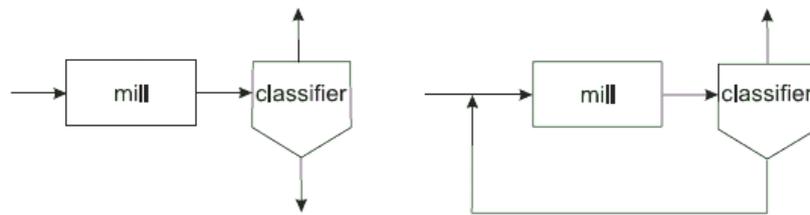


Fig. 1

In the first case the output material is simply separated in fractions with different particle sizes and the grinding process is actually not affected by the classifier. In the second case the classifier returns coarse material back to the mill feed and only the fine product is obtained at the output of the grinding circuit. In practice various types of interconnections between mills and classifiers are possible aimed at increasing the grinding efficiency of the overall process.

The main objectives of the grinding process include obtaining a desired particle size distribution in the final product without metal or other possible contamination, increasing the throughput of the grinding circuit and reducing the production cost of the overall process. To achieve these objectives various mathematical models and control methods are developed and applied in practice. In this paper we provide a brief survey of the basic principles in modeling of the grinding process and analyze several control strategies applied in the design of a control structure and the implementation of an appropriate process control method.

## II. Modeling of the process

### II.1. Basic fragmentation mechanisms

The main idea in modeling all comminution processes, including the grinding process, is to obtain mathematical relations between the size of the feed and the size of the product. Particles in the feed repetitively reduce their size due to the imparting energy of the grinding media which disrupts their binding forces. The size reduction is a result of the following three basic fragmentation mechanisms [25, 46].

– Abrasion occurs when local low intensity stresses are applied and the result is fine particles taken from the surface of the mother particle and particles of size close to the size of the mother particle (Fig. 2a).

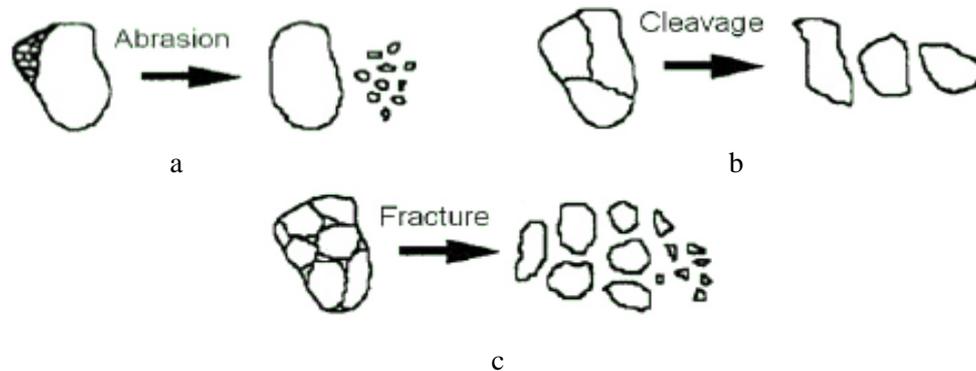


Fig. 2

– Cleavage of particles occurs when slow and relatively intense stresses are applied (compression) which produce fragments of size 50-80 % of the size of the initial particle (Fig. 2b).

– Fracture is a result of rapid applications of intense stresses (impact) which produce fragments of relatively small sizes with a relatively wide particle size distribution (Fig. 2c).

In practice the three different mechanisms never occur alone and the process of particle size reduction involves all of them with possible predominance depending on the type of the mill, the operating conditions and the type of the material being ground.

Several basic concepts are commonly used in modeling of the grinding process. The starting material naturally consists of particles which differ significantly in size which makes it necessary to define different size classes. Standard sieves are used to determine quantitatively the size of the particles in each size class. In the theory of breakage of solids, the fragmentation process is decomposed into a series of steps consisting of two main operations [46]:

(i) selection of a fraction of the material to be broken,

(ii) breakage of the selected material producing a given distribution of fragment sizes.

These operations are characterized by two functions: the selection function  $S_i$ ,  $i=1, 2, \dots, n$ , and the breakage function  $b_{ij}$ ,  $n \geq i \geq j \geq 1$ , where  $n$  is the number of the size classes. The selection function  $S_i$ , (also called probability of breakage or specific breakage rate [23, 45]) represents the probability for a particle of size of  $x_i$  to be broken at a given fragmentation step, where  $x_i$  is the lower limit of the particle size class  $i$ . Thus,  $S_1, S_2, \dots, S_n$  are the mass fractions of the material in each size class that are selected for size reduction. The breakage function  $b_{ij}$  (also

known as the distribution function [45, 46]) describes the distribution of fragment sizes obtained after a breakage of particles of size  $x_j$ . Thus,  $b_{1j}, b_{2j}, \dots, b_{nj}$  are the mass fractions of particles in size classes 1, 2,  $\dots$ ,  $n$  after a breakage of particles in size class  $j$ .

The mechanism of breakage is illustrated in [23] by a diagram shown in Fig. 3. The left column of the figure shows the size distribution of the feed. The application of forces on the particles in different size classes is shown by solid arrows and the movement of fragments to the same or lower size of classes is indicated by dotted arrows. The breakage functions are shown in the third column and the fourth column contains products obtained after a number of size reductions. During the process, the mass of feed in size class 1 is distributed in the lower size classes. At certain time, the mass fraction in size 1 will disappear as the particles are broken and distributed in the smaller sizes. It should be noted however, that the total mass remains constant.

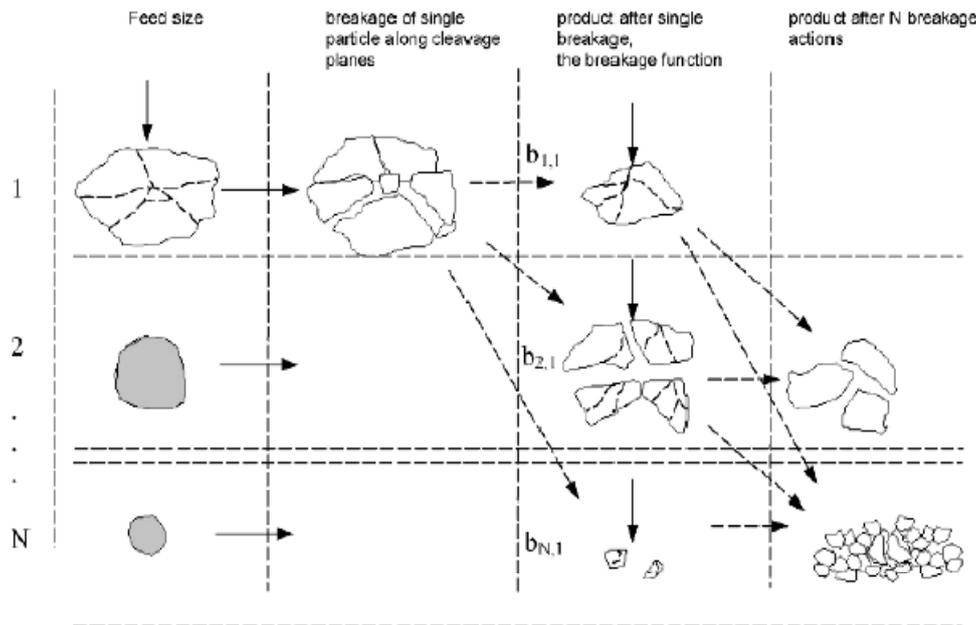


Fig. 3

The selection function and the breakage function are basic parameters in the modeling of all comminution processes. Three types of models are commonly accepted in literature [23, 45]: matrix, kinetic and energy models. A general principle in the development of each model is to establish mass balance or energy balance equations relating to the mass components or the energy involved in the process. The matrix type models are used when the size reduction is predominantly considered as a discrete process with each discrete step including the three operations selection-breakage-classification. For modeling of the batch grinding in continuous steady state mills, the size reduction is treated as a continuous process and the corresponding mathematical models take into account the time dependant

parameters of the process. In this case the kinetic and energy type of models are most frequently used.

## II.2. Kinetic and energy models

Several basic assumptions are made in modeling of the grinding process in various types of mills and milling circuits. Most frequently, it is assumed that the mill's content is uniform and thoroughly mixed by the rotation of the mill and the movement of the grinding media [44]. In this case the model is known as a perfectly mixed model. In some cases the mill's charge is considered to be perfectly mixed in the radial direction and only partially mixed in the axial direction. Another important assumption is that the particles of different sizes are broken in a similar way (normalized breakage) and that no agglomeration processes take place during the size reduction [23, 45].

The kinetic models of the grinding process are based on mass-balance equations describing the process in the different size intervals. Assuming that the mill is perfectly mixed in the radial direction and partially mixed in the axial direction, a kinetic model of second order is given in [45] in the form

$$(1) \quad \frac{dw_i(l,t)}{dt} = -S_i w_i(l,t) + \sum_{j=1}^{i-1} S_j b_{ij} w_j(l,t) + D_i \frac{d^2 w_i(l,t)}{dl^2} - u_i \frac{dw_i(l,t)}{dl},$$

where

$t$  is the grinding time;

$l$  – the space coordinate in the axial direction;

$w_i(l,t)$  – the mass fraction of material in the  $i$ -th size class;

$b_{ij}$  – the breakage function;

$S_i$  – the selection function;

$D_i$  – a mixing coefficient;

$u_i$  – the velocity of convective transport of particles in the axial direction.

The left hand side of equation (1) represents the variation of the mass fraction of the material in size class  $i$  within a time interval  $[t, t+dt]$ . The first and second term in the right hand side represent the mass of disappearing and appearing particles in this class, respectively. The third term describes the axial dispersion and the last term represents the convective transport of particles in the axial direction with velocity  $u_i$ . The differential equation (1) has the following boundary conditions:

$$(2) \quad w_i(l,0) = f_i(l),$$

$$(3) \quad w_i(l,t) = u_i w_i(l,t) - D_i \frac{dw_i(l,t)}{dl} \quad \text{for } l=0,$$

$$(4) \quad \frac{dw_i(l,t)}{dl} = 0 \quad \text{for } l=L,$$

where  $f_i(l)$  is the mass fraction of the feed in size class  $i$  and  $L$  is the length of the mill.

Equation (1) with conditions (2)-(4) represents the basic kinetic model of the process. Depending on the specific operational conditions of the mill, different variants of this model are also known. Most frequently, the perfectly mixed model is used under the assumption that the charge is thoroughly mixed and uniform in both radial and axial direction. In this case, the third and fourth term in (1) can be neglected and the grinding kinetics is described in the form [3, 25, 46]

$$(5) \quad \frac{dw_i(t)}{dt} = -S_i w_i(t) + \sum_{j=1}^{i-1} S_j b_{ij} w_j(t) .$$

That equation can be written in a matrix form as

$$(6) \quad \frac{dw(t)}{dt} = (B - I)S w(t),$$

where  $S$  is a diagonal matrix with diagonal elements  $S_i$ ,  $i = 1, 2, \dots, n$ ,  $B$  is a lower triangular matrix with elements  $b_{ij}$ ,  $n \geq i > j \geq 1$ ,  $w(t)$  is a vector with elements  $w_i(t)$ ,  $i = 1, 2, \dots, n$ , and  $I$  denotes the identity matrix. Matrix  $(B - I)S$  in (6) has a lower triangular form with diagonal elements  $-S_1, -S_2, \dots, -S_n$ . Under the assumption that functions  $b_{ij}$  and  $S_i$  are known and time independent, the solution of (6) is given by

$$(7) \quad w(t) = \exp[(B - I)St]w(0),$$

where  $\exp[(B - I)St]$  is the matrix exponent and  $w(0)$  is the vector of initial conditions with elements equal to the mass fractions of the feed in the respective class sizes. The explicit formulas for  $w_i(t)$ ,  $i = 1, 2, \dots, n$ , are known as the Reid solution to the batch grinding equation and can be found in [3, 23].

A cumulative form of equation (5) is also used in modeling of the grinding process, i.e.

$$(8) \quad \frac{dR_i(t)}{dt} = -S_i R_i(t) + \sum_{j=1}^{i-1} R_j(t) [S_{j+1} B_{ij+1} - S_j B_{ij}],$$

where  $B_{ij} = \sum_{k=i+1}^n b_{kj}$  is the cumulative breakage function and  $R_i(t) = \sum_{j=1}^i w_j(t)$  is the cumulative mass fraction of particles with size greater than  $x_i$ , the lower limit of the particle size class  $i$ .

Finding the solutions of equations (5) and (8) pre-supposes a preliminary knowledge of the breakage and selection functions  $b_{ij}$  and  $S_i$ . However, for a particular process these functions are not known apriori and they are usually determined by experimental tests and a consecutive treatment and estimation of the experimental results. Different methods for determination of these functions and some typical graph plots of breakage functions are given in [23, 25, 46]. Approximate solutions of the cumulative grinding equation (8) in an explicit form are also shown in [3].

Mathematical models based on energy-balance equations are also used in order to describe the grinding process. A linear model which is analogous to (5) is

developed in [21] where the batch grinding kinetics is expressed in terms of the specific energy as an independent variable instead of the grinding time. The authors of this reference conducted a series of experiments accurately measuring the specific energy consumed by the ball mill under various operating conditions and different ground material. An analysis of the results obtained in dry milling conditions shows that the size-discretized breakage rate functions are proportional to the specific energy input to the mill and that the breakage distribution functions can be considered invariant [21]. In this case an energy-balance equation modeling the grinding process can be given in the form

$$(9) \quad \frac{dw_i(\bar{E})}{d\bar{E}} = -S_i^E w_i(\bar{E}) + \sum_{j=1}^{i-1} S_j^E b_{ij} w_j(\bar{E}),$$

where  $\bar{E}$  is the specific energy input to the mill and  $S_i^E$  is the energy-normalized breakage rate parameter defined as

$$(10) \quad S_i^E = \frac{S_i}{P/W}.$$

In equation (10)  $P$  is the power input to the mill and  $W$  is the mass of the feed material in the mill.

A simple analysis and comparison between the mass-balance model given by (5) and the energy-balance model in the form (10) is as follows. Both equations (5) and (9) represent linear models which are relatively simple and easy to use in studying the first-order breakage kinetics of the process. The model (5) is thoroughly studied and solutions of the differential equation in explicit forms are obtained in literature under different assumptions and degrees of approximation. In general, a solution of (5) describes the change in particle size distribution of the ground material as a function of the grinding time. On the other hand, the model (9) shows that the breakage kinetics can be accurately analyzed in terms of the consumed specific energy instead of time. Furthermore, the power input to the mill can be accurately measured which makes it possible to use the measured data as an efficient control parameter in the process. It is pointed out in [21] that the energy model can also be very useful with regard to mill scale-up and the analysis of other types of comminution systems such as roll mills. Except for the linear models, it should be noted that a subject of particular interest is the development of more precise and complicated mathematical models of the grinding process including nonlinearities and time-dependant selection and breakage functions [7, 20]. Computer simulations based on the discrete element method [31, 32, 38] are also widely used in studying dynamical properties of the milling process.

### III. Process control methods

The control of a grinding circuit is a difficult task due to many factors such as nonlinear and undetermined character of the process, inaccuracies in the mathematical model, the presence of interacting process variables with substantially different dynamics, the influence of unmeasured disturbances and large time delays, rough operating conditions and inability to use precise and reliable sensors. On the

other hand, an efficient control of the process is of great importance for increasing the throughput of the circuit and quality of the final product as well as for a significant reduction of the production costs.

### III.1. Process variables and characteristics

From a control point of view, a ball mill grinding circuit represents an interconnected multivariable system with strong interactions among process variables. A typical structure of a closed-loop circuit for wet grinding consists of a ball mill, sump and classifier [10, 13, 33, 39] and it is schematically shown in Fig. 4.

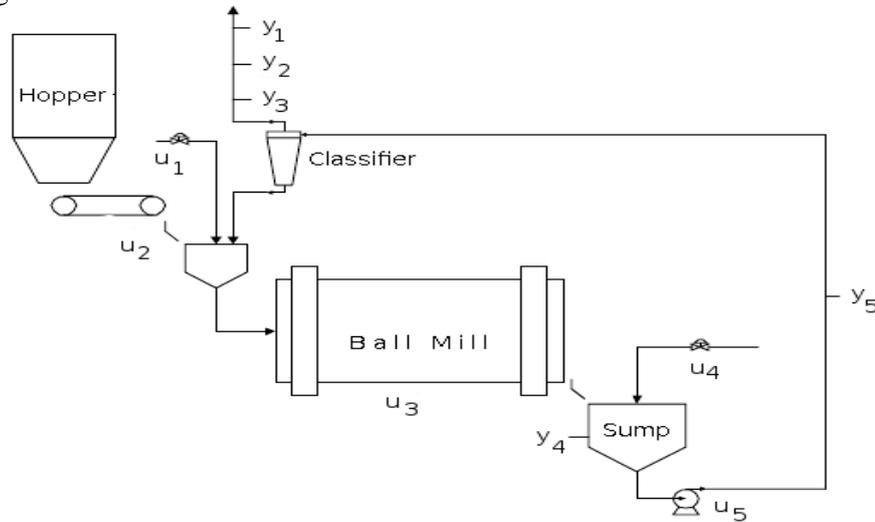


Fig. 4

The process input variables in the figure are:  $u_1$  – mill feed water flow rate,  $u_2$  – fresh ore feed rate,  $u_3$  – mill critical speed fraction,  $u_4$  – sump dilution water flow rate and  $u_5$  – sump discharge flow rate. The values of these variables can be manipulated in order to control the output variables:  $y_1$  – product mass fraction with size of particles less than a given value,  $y_2$  – product solids concentration,  $y_3$  – product flow rate,  $y_4$  – slurry level in the sump,  $y_5$  – sump solids concentration. The most important disturbances to the process are ore hardness changes and feed size variations. An input-output model of the process can be written in a vector matrix form:

$$(11) \quad \begin{bmatrix} y_1(s) \\ \vdots \\ y_5(s) \end{bmatrix} = \begin{bmatrix} G_{11}(s) & \dots & G_{15}(s) \\ \vdots & \ddots & \vdots \\ G_{51}(s) & \dots & G_{55}(s) \end{bmatrix} \cdot \begin{bmatrix} u_1(s) \\ \vdots \\ u_5(s) \end{bmatrix},$$

where  $G_{ij}(s)$  is the transfer function relating the  $i$ -th input and  $j$ -th output for  $i, j = 1, \dots, 5$ . The transfer functions are usually experimentally determined by applying step changes in the input and measuring the output responses. In order to obtain correct results usually a sufficiently large number of experiments should be carried out. Also, depending on the grinding circuit configuration and equipment,

different sets of input and output variables can be used in the control design [8, 33, 40].

The process control in a ball mill grinding circuit faces severe difficulties due to the following well known characteristics:

- the process is nonlinear with immeasurable disturbances and unmodelled dynamics;
- there are strong interconnections among variables so that each input variable interacts with multiple output variables;
- the time constants of the process have values in a wide range and there are significant time delays in some input-output pairs;
- the system model contains a number of integrators;
- the process parameters vary in time as the circuit ages;
- there are technological constraints on the manipulated and controlled variables;
- the measurements are unreliable and noisy.

The main control objectives are as follows. At the first place, it is necessary to maintain a stable operation at fixed set-points of the output variables. Within this objective, it is most important to maintain a stable product size distribution measured as a percentage of the output material with size of particles less than a given value. Alternatively, the control objective can be formulated as an optimization problem including a performance criterion which is to be optimized subject to certain constraints. It can be either the maximization of the grinding circuit throughput or minimization of the production costs. Various constraints in the optimization problem are the minimal or maximal values of the input, output and internal process variables which represent equipment limits, safe operation requirements or environmental regulations.

Due to the inherent process characteristics and constraints, the control design for a grinding circuit appears to be a challenging problem for most of the control methods applied in practice. In the rest of this section, two of the most frequently used control approaches are briefly described and an analysis of their main advantages and drawbacks is presented.

### III.2. Decentralized control

According to the results from a statistical study [48], the decentralized control approach is most frequently used in ball mill grinding circuits. The main advantage of this approach is the distributed control structure and its easy implementation in practice. Such a structure may consist of several single-input single-output control loops involving different process variables. The schematic diagram of a two-input two-output system is shown in Fig. 5, where the transfer functions of the two open loops with interconnections are denoted by  $G_{ij}(s)$ ,  $i, j = 1, 2$ , and the transfer functions of the controllers are  $G_{C1}(s)$  and  $G_{C2}(s)$ .

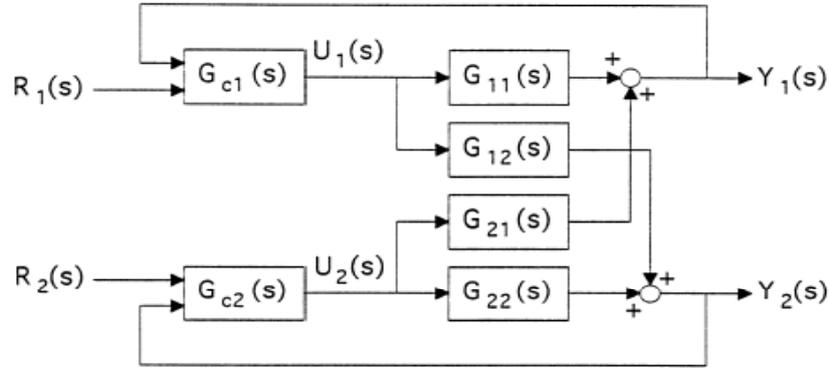


Fig. 5

A frequency-based method for the design of extended PID controllers is described in [35] where the transfer functions  $G_{C1}(s)$  and  $G_{C2}(s)$  are determined in the form

$$(12) \quad G_{C1}(s) = G_{11} - \frac{G_{12}(s)G_{21}(s)G_{C2}(s)}{1+G_{C2}(s)G_{22}(s)}, \quad G_{C2}(s) = G_{22} - \frac{G_{12}(s)G_{21}(s)G_{C1}(s)}{1+G_{C1}(s)G_{11}(s)}.$$

From (12) it is seen that the two controllers are interdependent due to the system interconnections and hence, the individual tuning of each controller depends on the parameters of the other. To overcome this problem, a procedure to determine the exact values of  $G_{C1}(s)$  and  $G_{C2}(s)$  is outlined in [35] by using a supplementary information from the system closed-loop specifications.

An important problem in the design of a distributed grinding process control is the choice of appropriate pairs of manipulated and controlled variables. When the input-output pairs are not properly selected, undesirable interactions between the control loops take place resulting in a poor control performance. Two most common variants of variable pairings in a grinding circuit with a sump and a hydrocyclone classifier are shown in Table 1 [13].

Table 1

Pairing	Controlled Variable	Manipulated Variable
I	Particle size hydrocyclone overflow	Sump water dilution rate
	Hydrocyclone feed rate	Fresh solids feed (& dilution rate)
II	Particle size hydrocyclone overflow	Fresh solids feed (& dilution rate)
	Hydrocyclone feed rate	Sump water dilution rate

Analysis of the dynamic responses to a set point change of the product size shows that there is a significant interaction between the control loops in both cases of pairing I and pairing II. Due to the strong interactions among the variables, a proper choice of the manipulated-controlled pairs should be based on information and analysis of both the steady state and the dynamic behavior of the control loops. In case of more than two input and two output variables, the existing control theory offers a valuable tool for selection of manipulated-controlled variable pairing which is known as the relative gain technique [6]. The method involves construction of a relative gain array associated with the system and the choice of pairs is based on the analysis of its structure and properties.

Another frequently used approach to compensate the system interactions in a decentralized control structure consists of including additional decoupling controllers between the control loops of the system. An illustration of a system with two control loops and decouplers is shown in Fig. 6, where  $D_{ij}(s)$ ,  $i, j = 1, 2$ , are transfer functions of the decoupling controllers.

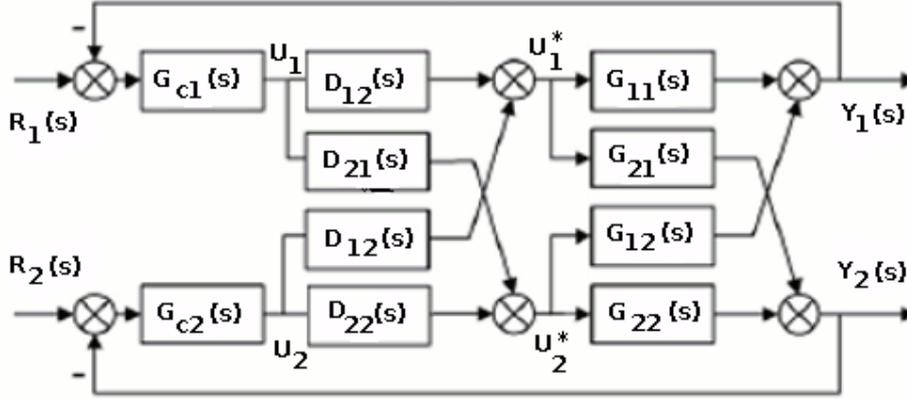


Fig. 6

The system is described by the vector-matrix equations  $U = G_C(R - Y)$ ,  $U^* = DU$ ,  $Y = GU^*$  which give the input-output equation

$$(13) \quad Y = GDG_C(R - Y),$$

where

$$(14) \quad Y = \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix}, G = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}, D = \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix}, G_C = \begin{bmatrix} G_{C1} & 0 \\ 0 & G_{C2} \end{bmatrix}, R = \begin{bmatrix} R_1 \\ R_2 \end{bmatrix}.$$

Since  $G_C$  is a diagonal matrix, the aim is to determine the transfer functions of decoupling controllers such that  $GD$  is a diagonal matrix, i.e.

$$(15) \quad GD = \begin{bmatrix} X_1 & 0 \\ 0 & X_2 \end{bmatrix}.$$

From (15), we have

$$(16) \quad D = G^{-1} \begin{bmatrix} X_1 & 0 \\ 0 & X_2 \end{bmatrix} = \frac{1}{\det(G)} \begin{bmatrix} G_{22}X_1 & -G_{12}X_2 \\ -G_{21}X_1 & G_{11}X_2 \end{bmatrix}.$$

If  $X_1$  and  $X_2$  are determined as  $X_1 = \det(G) / G_{22}$  and  $X_2 = \det(G) / G_{11}$ , then the transfer functions of decoupling controllers are obtained from (16) in the form

$$(17) \quad D_{11}(s) = 1, D_{22}(s) = 1, D_{12}(s) = -\frac{G_{12}(s)}{G_{11}(s)}, D_{21}(s) = -\frac{G_{21}(s)}{G_{22}(s)}.$$

The above equations show that the decoupling controllers are independent on the forward path controllers  $G_{C1}(s)$  and  $G_{C2}(s)$ . This is an important advantage of the method as far as the controller modes of operation and tuning of  $G_{C1}(s)$  and  $G_{C2}(s)$  can be changed without loss of decoupling of the control loops.

### III.3. Multivariable control

Multivariable control methods generally dominate in industrial processes with strong interactions among process variables. In the control of ball mill grinding circuits, a multivariable approach based on Model Predictive Control (MPC) strategy is successfully used [26, 33, 39]. This approach has numerous industrial applications where the control is designed to drive the process from one constrained steady state to another. The main objectives of the MPC system are outlined in [37] as follows:

- prevent violation of input and output constraints;
- drive the controlled variables to their steady-state optimal values (dynamic output optimization);
- drive the manipulated variables to their steady-state optimal values using remaining degrees of freedom (dynamic input optimization);
- prevent excessive movement of manipulated variables;
- when signals and actuators fail, control as much of the plant as possible.

The MPC utilizes control structures incorporating an internal process model and a predictive control computation. The internal model is used to predict the future output of the process. The future actions of the manipulated variables are determined by solving a finite horizon optimization problem of minimizing the difference between the desired reference trajectory and the predicted output. The optimization problem is solved at each sampling interval and as a solution a sequence of future control actions is obtained. At the next sampling interval only the first control action is applied and the procedure is repeated. Reference [37] presents an extensive survey of control algorithms employing the MPC approach. An up-to-date review of the current practice and challenges of MPC can be found in [14] and an exposition of the theory of MPC is presented in a monograph [16].

One of the main MPC methods is based on the Dynamic Matrix Control (DMC) algorithm which was developed in the late 70's of the last century and it was initially intended for use in petroleum industry. Subsequently, control strategies utilizing the DMC algorithm have become a powerful tool for process control in various ball mill grinding circuits [8, 10, 39]. In [8] an implementation of the DMC algorithm is presented as it is shown in Fig. 7.

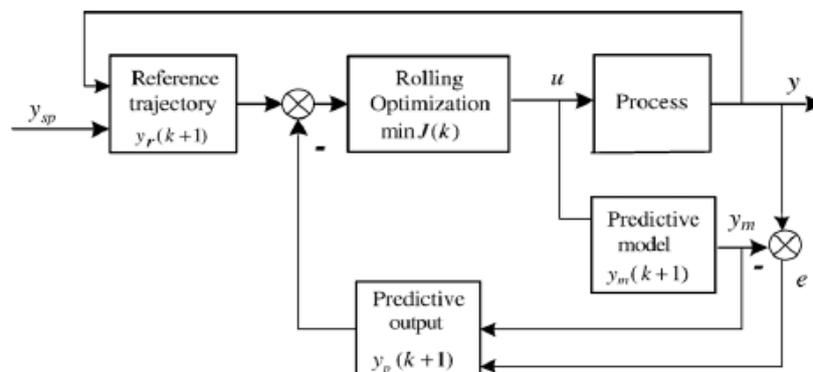


Fig. 7

If  $P$  denotes the length of output prediction horizon, then the predicted output vector

$$Y_p(k+1) = [y_p(k+1), \dots, y_p(k+P)]^T$$

is obtained as

$$(18) \quad Y_p(k+1) = Y_m(k+1) + G_0 E(k),$$

where  $Y_m(k+1) = [y_m(k+1), \dots, y_m(k+P)]^T$  is the model output,  $G_0$  is a coefficient matrix and

$$E(k) = Y(k) - Y_m(k)$$

is the difference between the process and model output vectors. The predicted control vector is in the form

$$\Delta U(k) = [\Delta u(k), \dots, \Delta u(k+M-1)]^T,$$

where  $M$  is the length of control prediction horizon. The values of  $\Delta U(k)$  are determined by a minimization of the following quadratic objective function [8]

$$(19) \quad J(k) = [Y_r(k+1) - Y_p(k+1)]^T Q [Y_r(k+1) - Y_p(k+1)] + \Delta U(k)^T R \Delta U(k),$$

where  $Q$  is an output weighting coefficient matrix and  $R$  is a control weighting coefficient matrix. Constraints on the process variables are given in the form

$$\begin{aligned} u_{\min} &\leq u(k) \leq u_{\max}, \quad k = 0, 1, \dots, M-1, \\ \Delta u_{\min} &\leq u(k) - u(k-1) \leq \Delta u_{\max}, \quad k = 0, 1, \dots, M-1, \\ y_{\min} &\leq y(k) \leq y_{\max}, \quad k = 0, 1, \dots, P-1. \end{aligned}$$

In applying MPC algorithm to ball mill grinding process, it is necessary to properly select the values of several important control parameters, such as the length of model output prediction horizon, the length of control prediction horizon, the weighting coefficients of output deviations and manipulated input variations. The length of output prediction horizon  $P$  is generally set long enough in order to capture the steady state behavior of the output. Concerning the control prediction horizon  $M$ , it is clear that an increasing of  $M$  will improve the control performance but the price will be an increased amount of computations. Generally,  $M$  is shorter than  $P$ . On the other hand, the desired output behavior and the control cost determine the choice of weighting coefficients in the objective function. If a tighter control of some output variable is desired, it can be achieved by a higher value of the respective weighting coefficient. Similarly, excessive variations in the manipulated variables can be suppressed by increasing the values of control weighting coefficients.

The surveyed simulation studies and practical applications of decentralized and multivariable control systems in ball mill grinding circuits reveal the following important characteristics of the two control strategies.

- Decentralized systems with local PI or PID controllers predominate in practical applications due to the simple control system implementation. The process variables are coupled in single-input single-output control loops, such that one manipulated variable is used for one controlled variable. The main problems in this approach are the choice of variable pairing and the decoupling of control loop

interactions by an appropriate tuning or by introducing decoupling controllers. On the other hand, the multivariable approach avoids these problems at the expense of a more complicated control system.

- Multivariable methods using MPC algorithms are becoming increasingly popular with successful implementations reported in the literature. Advantages of this approach are its capability to cope with the strong interactions among process variables and the possibility for a better control system tuning. However, the MPC requires an accurate process modeling at the design stage and substantially more complicated computations during the process control.

- Decentralized PID controllers normally give satisfactory results with respect to the steady state error and closed loop responses to small set point changes provided that a proper variable pairing is used and the coupling among control loops is taken into account. However, results from simulation studies show significant advantages of MPC as compared to decentralized PID controllers. In particular, MPC schemes explicitly take into account the technological constraints of the manipulated and controlled variables and allow for achieving optimal operating conditions of the control system. Also this approach successfully overcomes problems associated with unmeasured disturbances, time-delays and nonlinearities of the process.

- The choice of a control method and control system structure for a particular ball mill grinding process should normally take into account certain economic value indicators. It turns out that it is difficult to estimate the potential economic profit in using more complex control structures and computations instead of simplified control schemes. Furthermore, an investigation [47] estimates the impact of particle size distribution and comes to a conclusion that a larger economic profit can be achieved due to the set point move to a better operating point than due to a tighter process control. In general, the energy efficiency and economic assessment of the process control in industrial ball mills remains an open problem motivating future research activities and comprehensive studies [2, 19, 26, 27, 48].

We shall conclude this section by mentioning several other advanced techniques applied to the process control in ball mills. An approach involving simultaneous decoupling of interactions and closed loop pole assignment is proposed in [24]. A control system incorporating disturbance observer and MPC is designed in [50]. Innovative control platforms are described in [43] and a comparison of multivariable PI, fuzzy and model predictive control is presented in [40]. A survey of adaptive control methods and their application to the grinding process is given in [34]. Simulation results with expert system based control, supervisory control and observer based control are reported in [9, 11, 12]. An integrated automation system for monitoring and grinding process control is designed in [41] and a neurocontrol approach is developed in [13].

## IV. Conclusion

The existing mathematical models of the grinding process are developed on the basis of mass balance or energy balance equations which describe the particle size reduction of the ground material as a function of the grinding time or in terms of the consumed specific energy. In both cases the main parameters of the model are selection and breakage functions which are generally not known a priori and their determination requires additional experimental studies. A subject of particular interest in the process modeling is the development of more precise and complicated mathematical models including nonlinearities and time-variant selection and breakage functions. Concerning the process control, decentralized and multivariable control methods predominate in the surveyed simulation results and practical applications. Nevertheless, theoretical and simulation studies of control systems employing advanced control techniques, such as adaptive, expert system and fuzzy logic control are gaining increased interest. Important open problems which motivate further research activities in this area are related with the energy efficiency of the ball mill and the economic assessment of the process control.

## References

1. Ajaal, T., R. W. Smith, W. T. Yen. The Development and Characterization of a Ball Mill for Mechanical Alloying. – Canadian Metallurgical Quarterly, Vol. **41**, 2002, No 1, 7-14.
2. Bauer, M., I. K. Craig. Economic Assessment of Advanced Process Control – A Survey and Framework. – Journal of Process Control, Vol. **18**, 2008, 2-18.
3. Berthiaux, H., C. Varinot, J. Dodds. Approximate Calculation of Breakage Parameters from Batch Grinding Tests. – Chemical Engineering Science, Vol. **51**, 1996, No 19, 4509-4516.
4. Bond, F. C. Crushing and Grinding Calculations. Part I. – British Chemical Engineering, Vol. **6**, 1961, No 6, 378-385.
5. Bond, F. C. Crushing and Grinding Calculations. Part II. – British Chemical Engineering, Vol. **6**, 1961, No 8, 543-548.
6. Bristol, E. G. On a New Measure of Interaction for Multivariable Process Control. – IEEE Trans. Auto. Control, AC-11, Vol. **1**, 1966, 133-134.
7. Capece, M., E. Bilgili, R. Dave. Identification of the Breakage Rate and Distribution Parameters in a Non-Linear Population Balance Model for Batch Milling. – Powder Technology, Vol. **208**, 2011, 195-204.
8. Chen, X. S., Q. Li, S. M. Fei. Constrained Model Predictive Control in Ball Mill Grinding Process. – Powder Technology, Vol. **186**, 2008, 31-39.
9. Chen, X. S., S. H. Li, J. Y. Zhai, Q. Li. Expert System Based Adaptive Dynamic Matrix Control for Ball Mill Grinding Circuit. – Expert Systems with Applications, Vol. **36**, 2009, 716-723.
10. Chen, X. S., J. Y. Zhai, S. H. Li, Q. Li. Application of Model Predictive Control in Ball Mill Grinding Circuit. – Minerals Engineering, Vol. **20**, 2007, 1099-1108.
11. Chen, X. S., Q. Li, S. M. Fei. Supervisory Expert Control for Ball Mill Grinding Circuits. – Expert Systems with Applications, Vol. **34**, 2008, 1877-1885.
12. Chen, X. S., J. Yang, S. H. Li, Q. Li. Disturbance Observer Based Multi-Variable Control of Ball Mill Grinding Circuits. – Journal of Process Control, Vol. **19**, 2009, 1205-1213.
13. Conradie, A. E., C. Aldrich. Neurocontrol of a Ball Mill Grinding Circuit Using Evolutionary Reinforcement Learning. – Minerals Engineering, Vol. **14**, 2001, Issue 10, 1277-1294.

14. Darby, M. L., M. Nikolaou. MPC: Current Practice and Challenges. – Control Engineering Practice, Vol. **20**, 2012, 328-342.
15. Deniz, V. A Study on the Specific Rate of Breakage of Cement Materials in a Laboratory Ball Mill. – Cement and Concrete Research, Vol. **33**, 2003, 439-445.
16. Ding, B. C. Modern Predictive Control. Taylor and Francis Group, 2010.
17. DOERING International GmbH.  
<http://www.doering-sinn.de/en/grinding-media.html>
18. Erdem, A. S., S. L. Ergun. The Effect of Ball Size on Breakage Rate Parameter in a Pilot Scale Ball Mill. – Minerals Engineering, Vol. **22**, 2009, 660-664.
19. Fuerstenau, D. W., A. Z. M. Abouzeid. The Energy Efficiency of Ball Milling in Comminution. – Int. Journal Miner. Process., Vol. **67**, 2002, 161-185.
20. Fuerstenau, D. W., A. De, P. C. Kapur. Linear and Nonlinear Particle Breakage Processes in Comminution Systems. – Int. Journal Miner. Process, Vol. **74S**, 2004, 317-327.
21. Fuerstenau, D. W., P. C. Kapur, A. De. Modelling Breakage Kinetics in Various Dry Comminution Systems. – KONA Powder and Particle Journal, Vol. **21**, 2003, 121-132.
22. Fuerstenau, D. W., J. J. Lutch, A. De. The Effect of Ball Size on the Energy Efficiency of Hybrid High-Pressure Roll Mill/Ball Mill Grinding. – Powder Technology, Vol. **105**, 1999, 199-204.
23. Gupta, A., D. Yan. Mineral Processing Design and Operations: An Introduction. Elsevier, May 2006. 718 p.
24. Hauksdóttir, A. S., M. Ierapetritou, S. L. J. Jounela, U. Zühlke. A Decoupling Grinding Circuit Control System. – In: 15th IFAC Triennial World Congress, Barcelona, Spain, 2002.
25. Hennart, S. L. A., W. J. Wildeboer, P. van Hee, G. M. H. Meesters. Identification of the Grinding Mechanisms and Their Origin in a Stirred Ball Mill Using Population Balances. – Chemical Engineering Science, Vol. **64**, 2009, 4123-4130.
26. Hodouin, D. Methods for Automatic Control, Observation, and Optimization in Mineral Processing Plants. – Journal of Process Control, Vol. **21**, 2011, 211-225.
27. Hodouin, D., S-L. Jamsa-Jounela, M. T. Carvalho, L. Bergh. State of the Art and Challenges in Mineral Processing Control. – Control Engineering Practice, Vol. **9**, 2001, 995-1005.
28. Kotake, N., K. Daibo, T. Yamamoto, Y. Kanda. Experimental Investigation on a Grinding Rate Constant of Solid Materials by a Ball Mill – Effect of Ball Diameter and Feed Size. – Powder Technology, Vol. **143/144**, 2004, 196-203.
29. Kotake, N., M. Kuboki, S. Kiya, Y. Kanda. Influence of Dry and Wet Grinding Conditions on Fineness and Shape of Particle Size Distribution of Product in a Ball Mill. – Advanced Powder Technology, 2010, doi:10.1016/j.apt.2010.03.015.
30. Kuzev, L., T. Penchev, D. Karastoyanov. New Shape Milling Bodies for Ball Mills. – Problems of Engineering Cybernetics and Robotics, Vol. **61**, 2009, 11-19.
31. Mishra, B. K., C. Thornton. An Improved Contact Model for Ball Mill Simulation by the Discrete Element Method. – Advanced Powder Technology, Vol. **13**, 2002, No 1, 25-41.
32. Mishra, B. K. A Review of Computer Simulation of Tumbling Mills by the Discrete Element Method. Part II – Practical Applications. – Int. Journal Miner. Process., Vol. **71**, 2003, 95-112.
33. Muller, B., P. L. de Vaal. Development of a Model Predictive Controller for a Milling Circuit. – Journal of the South African Institute of Mining and Metallurgy, Vol. **100**, 2000, No 7, 449-453.
34. Najim, K., D. Hodouin, A. Desbiens. Adaptive Control: State of the Art and an Application to Grinding Process. – Powder Technology, Vol. **82**, 1995, 59-68.
35. Pomerleau, A., D. Hodouin, A. Desbiens, E. Gagnon. A Survey of Grinding Circuit Control Methods: From Decentralized PID Controllers to Multivariable Predictive Controllers. – Powder Technology, Vol. **108**, 2000, 103-115.
36. Powell, M. S., I. Smit, P. Radziszewski, P. Cleary, B. Rattray, K. Eriksson, L. Schaeffer. The Selection and Design of Mill Liners. – In: Advances in Comminution. S. K. Kawatra, Ed. ISBN-13:978-0-87335-246-8, Society for Mining, Metallurgy, and Exploration, Inc., Colorado, USA, 2006, 331-376.

37. Qin, S. J., T. A. Badgwell. A Survey of Industrial Model Predictive Control Technology. – Control Engineering Practice, Vol. **11**, 2003, 733-764.
38. Rajamani, R. K., B. K. Mishra, R. Venugopal, A. Datta. Discrete Element Analysis of Tumbling Mills. – Powder Technology, Vol. **109**, 2000, 105-112.
39. Ramasamy, M., S. S. Narayanan, C. D. P. Rao. Control of Ball Mill Grinding Circuit Using Model Predictive Control Scheme. – Journal of Process Control, Vol. **15**, 2005, 273-283.
40. Remes, A., J. Aaltonen, H. Koivo. Grinding Circuit Modeling and Simulation of Particle Size Control at Siilinjärvi Concentrator. – International Journal of Mineral Processing, Vol. **96**, 2010, 70-78.
41. Remes, A., J. Karesvuori, H. Pekkarinen, S-L. Jämsä-Jounela. Integrated Approach to Monitoring and Control of Mineral Grinding Processes. – In: 1st IFAC Workshop on Applications of Large Scale Industrial Systems, Finland, 2006.
42. Shi, F. Comparison of Grinding Media – Cylpebs Versus Balls. – Minerals Engineering, Vol. **17**, 2004, 1259-1268.
43. Smith, G. C., L. Jordaan, A. Singh, V. Vandayar, V. C. Smith, B. Muller, D. G. Hulbert. Innovative Process Control Technology for Milling and Flotation Circuit Operations. – Journal of the South African Institute of Mining and Metallurgy, May 2004, 353-365.
44. Tavares, L. M., R. M. D. Carvalho. Modeling Breakage Rates of Coarse Particles in Ball Mills. – Minerals Engineering, Vol. **22**, 2009, 650-659.
45. Toneva, P., W. Peukert. Modelling of Mills and Milling Circuits. – Handbook of Powder Technology. Vol. **12**. 2007, 874-911. Particle Breakage. A. Salman, M. Ghadiri, M. Hounslow, Eds. Elsevier, 2007. 1240 p.
46. Varinot, C., S. Hiltgun, M. N. Pons, J. Dodds. Identification of the Fragmentation Mechanisms in Wet-Phase Fine Grinding in a Stirred Bead Mill. – Chemical Engineering Science, Vol. **52**, 1997, No 20, 3605-3612.
47. Wei, D., I. K. Craig. Economic Performance Assessment of Two ROM Ore Milling Circuit Controllers. – Minerals Engineering, Vol. **22**, 2009, 826-839.
48. Wei, D., I. K. Craig. Grinding Mill Circuits – A Survey of Control and Economic Concerns. – In: Proceedings of the 17th World Congress of the International Federation of Automatic Control, Seoul, Korea, 6-11 July 2008, 1000-1005.
49. Wills, B. A., T. J. Napier-Munn. Mineral Processing Technology. Elsevier Science & Technology Books. 7th Edition. 2006. 450 p.
50. Yang, J., S. Li, C. Xi-Song, Q. Li. Disturbance Rejection of Ball Mill Grinding Circuits Using DOB and MPC. – Powder Technology, Vol. **198**, 2010, 219-228.
51. Zoz, H., R. Reichardt, J. S. Kim. Application and Design of Drum Mills, PM<sup>2</sup>TEC'2002. – In: Proceedings of the World Congress on Powder Metallurgy & Particulate Materials, 16-21 June 2002, Orlando, FL, USA.